Applying Reversible Jump MCMC to Bayesian Model of Crime and Punishment in England, with reference to the Old Bailey Courthouse

A thesis presented for the Degree of Master of Science in Statistics Department of Mathematics and Statistics, University of Canterbury

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By Julianne Shields

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Abstract

In this thesis, we have used the Reversible Jump MCMC algorithm to model annual counts of various crimes committed and punishments meted out based on the trial records of the Old Bailey Courthouse in London over the period of 1674 – 1913. A total of 223,248 cases were heard over the 240 years. Our Bayesian model allows us to estimate not only the average frequencies of various crimes and punishments over the years, but also do identify points of abrupt changes, which may have been due to changes in legislation, societal norms or economic situations. We proffer a possible explanation for some of these findings and explain their relevancy to our times.

Acronyms and Abbreviations

Included below is a list of the acronyms and abbreviations which have been used throughout this thesis.

- MH: Metropolis-Hastings
- MC: Monte Carlo Methods
- MCMC: Monte Carlo Markov Chain
- RjMCMC: Reversible jump Monte Carlo Markov Chain
- DIC: Deviance Information Criterion
- MA: Model Averaged
- CI: Credible Interval

1 Introduction

1.1 Crime and Punishment in England

It is self-evident that over any extended period of time in which a society exhibits changing values, many aspects of life would also adapt to change. Those changes would, in turn, be seen in societal phenomena, one such example being a fluctuating crime rate. There are many factors that could be attributed to a variation in aspects of crime and punishment, and some of these will be explored in this thesis for the specific case of London over the time of the Old Bailey Courthouse. One interesting aspect of crime and criminality is the effect that crime can have on an the assessment of threat level and feelings of safety at the level of an individual.

The opening lines of Beattie's *Policing and Punishment in London 1660-1750* clearly underline the notion that the greatest threat to the feeling of safety in London at the time of the Old Bailey was a high incidence of personal crime:

> There was a common perception in London in the late seventeenth and early eighteenth centuries that crime was a serious problem. The offences that caused the sharpest anxieties and triggered the strongest responses were those that threatened individual victims in their person or property [. . .] Beattie (2001a) (p. 1).

The explanations for changes in the rates of crime and the resulting punishments that are proposed by historians are far ranging and still under debate. One such suggestion for increased crime rates was attributed to war and economic hardship (Beattie 2001a). The number of potential factors that could influence the crime rate grows largely when considering more specific sub-groups of society, for example, the influence of the gin trade over the young middle-class, or the effect that "lewd women" would have over their male counterparts, who were defenceless against their corruption.

Beyond the perceptions surrounding crime in the greater London community and the social setting at the time of the Old Bailey, physical changes in the trial procedure also occurred, stemming from the Old Bailey Courthouse itself.

1.2 History of the Old Bailey Courthouse

The Old Bailey was "the central criminal court for the City of London and the County of Middlesex", and as such was the setting in which the majority of serious crimes and offences were heard and tried in the greater London area (Emsley, Hitchcock, and Shoemaker 2015a).

The Old Bailey Courthouse, built in 1673, was used for criminal trials until 1913. Over the 240 years that it was in use, changes occurred to both the crimes punishable and the punishments enforced, due to a variety of factors. The Old Bailey was built in place of the old medieval courthouse that was destroyed in the Great Fire of London, 1666, and is still situated in its original location in the Western part of London (Emsley, Hitchcock, and Shoemaker 2015b).

A wide variety of crimes were tried at the courthouse, which can be broken down into the categories of: Breaking the Peace, Damage to Property, Deception, Killing, Offences Against the King (or Queen), Sexual Offences, Theft, Theft with Violence and the more general catgeory of "other offences". The scope of the crimes that were attributed to each of the categories evolved over time, as did the severity and methods of the punishments that were ordered (Emsley, Hitchcock, and Shoemaker 2015a).

The Courthouse itself was remodelled several times over the years, adapting to the changing needs of the citizens and criminals and incorporating more modern fixtures as they were developed (Emsley, Hitchcock, and Shoemaker 2015b). These physical changes reflected the evolving practices of the trial procedures. The original structure of the building was open to the elements so as to hinder the spread of typhus from prisoners to others. The trials were witnessed by a diverse audience encompassing both the upper and lower classes. Notably, it is suggested that criminals were in attendance to form strategies for avoiding prosecution, should they be arrested, and furthermore, that jurors could be influenced by the persuasion of the crowds of commoners. In 1737, it is speculated that the reason for the remodelling of the courthouse was to limit the influence of the spectators. In 1774, further reconstructions obscured the public view of the courtroom, prevented prisoners from communicating with others, and reduced the number of spectators that could enter the courtroom hastily by constructing narrow corridors. In 1824, a second courtroom was added to address the increasing numbers of trials, which continued to grow dramatically in future years. In this way, the physical privacy in a courtroom, or lackthereof, varied over time to suit the changing opinions surrounding spectacle and decency, which also extended to the details that were shared with the greater public through published media.

1.3 The Press and Public Opinion

Although the trials themselves were often a public spectacle, the public attention was not limited to the courtroom. Beginning in the sixteenth century, there was an interest in learning about the actions and repercussions of criminals of note, which endured throughout the time leading up to the creation of the "Sessions Papers" of the Old Bailey around the 1670s (Beattie 2001a, 2). What began as irregular notices of the courtroom business, became a regular publication following each session, detailing a brief outline of all of the cases that were tried (Beattie 2001a, 3). As suggested by Beattie, the Sessions Papers became "fuller and more complete, even quasi-official" (2001a, 3). It is the records of these accounts that have informed the views of historians such as Beattie as to the nature of crime in London, and from which the "Old Bailey Proceedings Online" project was created (Hitchcock et al. 2018).

In addition to the publications which became a focal point for fear, the much more literal focal point of watching the punishments being administered featured heavily in the lives of Londoners (Edwards 2013). The final moments of prisoners were witnessed by many, as crowds gathered to watch hangings at the gallows (Edwards 2013). The prisoners were charged with crimes ranging from petty theft to murder, and the punishment of some prolific criminals were a particular draw, such as Catherine Wilson, the last woman to be publicly hanged in London (Blanco 2017). Wilson was a nurse who, according to Murderpedia, poisoned her victims after convincing them to allow her to be a benefactor in their will (2017). She was eventually convicted of one murder, however it was well accepted that she had several other victims, following the exhumation of the bodies of seven of her previous patients, all of which showed evidence of poisoning (Blanco 2017). It is estimated that a staggering 20,000 spectators gathered to watch her death in 1862 (Edwards 2013). The hangings were so well attended that it is reported that in 1807, 28 people were crushed to death following a loss of crowd control, and a problematic growth in popularity led to the discontinuation of public execution by 1868 (Edwards 2013). The rise and fall of the death penalty as a possible consequence for certain offences was noted to have had an affect on the overall crime rates (Emsley, Hitchcock, and Shoemaker 2015a).

1.4 Conscience and Morality

The range of crimes tried, the conditions under which a conviction could be garnered, and the possible punishments that could be administered changed over the time of the Old Bailey (Emsley, Hitchcock, and Shoemaker 2015a). One key example of the changing attitude towards crime and punishment lies in the treatment of sexual offences, and the ammendments to both common and statute law. In order for a defendant to be convicted of rape, it had to be proven specifically that penetration occured against the will of the female victim. Due to lack of substantiating evidence and a general unwillingness to risk a false guilty verdict, it is noted that the successful conviction rate for rape plummeted to as low as 5%, which is, in part, attributed to the severity of the punishment upon conviction. Rape was a capital offence until 1841, after which approximately half of the cases of rape brought before Old Bailey were successfully prosecuted. The difficulties around conviction of rape were somewhat mitigated by charging the accused with the lesser offence of "assault with intent to rape". Even more archaic from a twenty-first century perspective are the laws pertaining to homosexual relationships and sodomy. In order to convict a man of practicing sodomy, it was necessary to prove that both penetration and ejaculation had occurred, and furthermore, this had to be corroborated by two witnesses. In a similar manner, a reduced charge of "assault with sodomitical intent" was often sought. Aside from the changing punishments relating to the crimes tried, there was also a notable shift in the way that crimes were reported in the *Proceedings*. *The Proceedings* were an invaluble source of information both at the time of publication, and today.

The crimes tried at the Old Bailey were largely dealt with by the court, however there were a number of crimes that were instead handled by the church, through pleading "Benefit of Clergy".

"Benefit of Clergy: A right to be excluded from the death penalty" (Emsley, Hitchcock, and Shoemaker 2019b). From the middle ages onwards, the church was granted the responsibility and right to punish its own members, with the burden of proof placed upon defendants to verify their membership to the church changed over time (Emsley, Hitchcock, and Shoemaker 2019a). Until 1706, upon a claim of benefit of clergy, the judge for the trial would select a biblical extract that the convict had to recite. This practice was common enough that "Psalm 51" was colloquially known as the "neck verse"; a popular text choice (Beattie 2001b). The judge was able to manipulate the outcome of the test somewhat, by subjectively deciding which text was used, and the minimum literacy required (Emsley, Hitchcock, and Shoemaker 2019a).

In order to ensure that a defendant could claim benefit of clergy only once, the convict would be branded in the courtroom, following the session (Emsley, Hitchcock, and Shoemaker 2019a). The branding was primarily on the thumb, however, for a period of time between 1699 and early 1707, some thieves were branded on the face, in hopes that it would create a greater deterrent. Benefit of clergy was abolished in 1827, following a period of law reform, however the crimes under which a defendant was not permitted to claim the benefit, crimes deemed "non-clergyable", changed over the course of time.

1.5 The problem with Metropolis Hastings

The purpose of this thesis is to explore the Old Bailey Courthouse proceedings over time and gain new insights into the changing social dynamics of the population of Greater London through a statistical lens. The behaviour of the crime in London appears to change with time, and can be modelled as a Poisson process and evaluated using Bayesian methods. Of particular interest to us is the points at which the incidence of crime changes, which we investigate through breakpoint analysis. By examining the locations of these breakpoints, we can begin to draw parallels to significant historical events and changes in the judicial system - for instance whether or not the threat of capital punishment was effective in preventing serious crime, such as murder.

Metropolis Hastings (MH) is an algorithm commonly used in determining Bayesian models, and may be a first choice when considering this problem, however, there are significant limitations posed by this. The generalized sampling method proposed by Hastings is effective at determining posterior estimates through Monte Carlo (MC) methods, however, the dimension of the model must be fixed beforehand (1970). In the context of our research, there is not sufficient basis for the assumption of a particular number of breakpoints, and as such we needed to look to a more specialized method, namely the Reversible jump Markov Chain Monte Carlo algorithm (RjMCMC) proposed in Green's article, "Reversible jump Markov chain monte carlo computation and Bayesian model determination" (1995).

2 Data

The majority of the knowledge gathered by historians on the trials held at the Old Bailey Courthouse comes from documents called the *Proceedings of the Old Bailey*, known simply as the *Proceedings* (Shoemaker 2008). These documents, first published in 1674, were often seen as both an account of trials of interest to the greater public, as well as providing scandalous entertainment through the inclusion of "titillating details of prostitutes' interactions with their clients", for example (Shoemaker 2008). The data for this thesis was sourced by the Old Bailey Online project, who aim to maintain an accurate account of the *Proceedings* through digitization, and offer aggregated datasets depending on specific filters or keywords that are chosen (Emsley, Hitchcock, and Shoemaker 2018b). Details regarding the reliability of this data and its importance as held by historians are explored in the Discussion section in later chapters.

Contained in this dataset are accounts of the offences charged and punishments handed down, aggregated by year. Over the 240 years of the Old Bailey, 211,123 offences were heard, resulting in 169,239 punishments allocated to 252,509 defendants. The Old Bailey Online project divided the data into offence and punishment categories, which we have represented as nine categories of crime, and six categories of punishment. In addition, we analysed the data as total counts of offences and punishments, adding to a total of 17 subsets of data. Each of these subsets was considered in turn, and results under both MH and RjMCMC methods were gathered.

3 Methods

3.1 The Model

In this thesis, we modelled the number of events (offences or punishments) that occurred annually, with the following parameters:

$$\begin{aligned} k &= \text{number of breakpoints}, \\ h_1, \ldots, h_{k+1} &= \text{intensities or heights in the intervals}, \\ s_1, \ldots, s_k &= \text{breakpoint locations} \end{aligned}$$

In this way, we have a total of $2(k+1)$ parameters in our model. This leads to the obvious note that the number of parameters and the resulting dimension of the model parameter space is dependent on k, and as such we employed a generalized MH procedure conditional on integer values of k, as well as the more flexible RjMCMC procedure proposed by Green in order to address the challenges posed by the changing dimension (1995).

3.2 MH Generally

The MH algorithm generally proceeds as follows (Hastings 1970). Given the likelihood $p(y|\theta)$, the prior $p(\theta)$, and the current value of the parameter $\theta = \theta_0$:

1. Choose new θ' from the proposal distribution
2. Evaluate the acceptance ratio based on the current state and the proposed state
3. Accept the proposed state θ' with probability α, or reject the proposal.

For our model, the RjMCMC procedure was carried out as per Green's article, "Reversible jump Markov chain monte carlo computation and Bayesian model determination", apart from a few adjustments that were made to better suit the data (1995). These adjustments are noted and addressed as they arise in describing the procedure. Under this framework, at each iteration one of the following step types is chosen:

1. H Height change - A change is made to the height, h_j, of a randomly chosen interval. The probability of H is given by η_k
2. P Position change - A change is made to the boundaries of a randomly chosen interval, s_j. The probability of P is given by π_k
3. B Birth step - A new breakpoint is created at a randomly chosen location, increasing the dimensionality of the model. The probability of B is denoted by b_k
4. D Death step - A randomly chosen interval "dies", decreasing the dimensionality of the model. The probability of D is d_k.

At each iteration, the algorithm selects one of these transitions and the associated updated parameters are proposed, which are accepted with probabilities detailed in the proceeding section. The aforementioned probabilities associated with the selection of each transition are follows:

For each iteration, the probability of a given step only depends on k, the number of current breakpoints. It is self evident that the step transition probabilities must total 1, that is:

$$\eta_k + \pi_k + b_k + d_k = 1, \qquad \text{for } k \in \{0, 1, \ldots, k_{\max}\} \tag{1}$$

Where $k_{\max}$ is specified, limiting the number of breakpoints.

For the situation $k = 0$, it is defined that $d_0 = \pi_0 = 0$, which prevents the algorithm from proposing a "death" to the only interval, and does not allow the boundaries of the interval to be changed, as they are already fixed $[0, L]$. For $k = k_{\max}$, a similar restriction is put in place, and $b_{k_{\max}} = 0$. This stops the dimension from increasing beyond a fixed threshold. For this analysis, we have followed the example of Green (1995), and aim to propose birth and death steps as often as possible without compromising algorithm performance. This was achieved by maximizing the constant, c, such that:

$$b_k = c \min\{1,\, p(k+1)/p(k)\} \tag{2}$$

$$d_{k+1} = c \min\{1,\, p(k)/p(k+1)\} \tag{3}$$

subject to

$$b_k + d_k \leq 0.9,\ k \in \{0, 1, \ldots, k_{\max}\} \tag{4}$$

Apart from the probabilities at the boundaries of the parameter space of k, the probability of selecting a height or position change is balanced, that is:

$$\eta_k = \pi_k \text{ for } k \neq 0$$

We now proceed to describe the steps.

3.3 The Preliminaries

In this chapter we generally review the procedure outlined in Green, while clarifying some derivations which were not made explicit in the paper (1995).

Let Y_t be the count in year t, where $t = 1, 2, ..., L$, and L is the last year of observation. Then, Y_t can be considered as a Poisson distributed variable, $Y_t|\lambda_t \sim \text{Pois}(\lambda_t)$, where $\lambda_t = h_j$ is the mean intensity over the interval $s_{j-1} < t \leq s_j,\ j \in \{1, 2, ..., k+1\}$. An interval is defined as the time between two boundaries, $s_{j-1} < t \leq s_j$ where $s_0 = 0$, and $s_{k+1} = L$.

The model thus has the following parameters:

$$\begin{aligned} k &= \text{number of breakpoints}, \\ h_1, \ldots, h_{k+1} &= \text{intensities (also referred to as heights) in the intervals}, \\ s_1, \ldots, s_k &= \text{breakpoint locations} \end{aligned}$$

In order to set-up a Bayesian model, we need to assign priors, which we do as follows:

For the number of breakpoints, k, we define a discrete uniform distribution between 0 and a maximum number, $k_{\max}$, that is fixed *a priori.*

$$p(k) = \begin{cases} \frac{1}{k+1} & \text{for } k \in [0, k_{\max}] \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

It should be noted that this differs from the more informative Poisson choice in Green. For the purposes of this analysis, we found that the number of breakpoints was not well enough informed to propose a Poisson prior, and after some preliminary testing, it was decided that $k_{\max}$ should be restricted to 100. The basis for this choice is that choosing a prior which favours a small number of breakpoints may dominate in the posterior, while a prior that allows for too many breakpoints does not make sense on the scale of this data. The data was aggregated according to year, as mentioned in the Data, so while breakpoints can theoretically occur at any time on $[0, L]$, there is not enough support over the Time domain to inform a large number of intervals.

For heights, h_j, we use a Gamma prior distribution, with probability density function:

$$f(h; \alpha, \beta) = \frac{\beta^\alpha h^{\alpha-1} e^{-\beta h}}{\Gamma(\alpha)}, \tag{6}$$

where $\Gamma(\alpha)$ is the usual Gamma function. We choose $\alpha = 0.01, \beta = 0.01$.

For the locations of the breakpoints, s_j, we again take Green's suggestion, and propose that the prior distribution is defined as the even-numbered order statistics of $2k + 1$ points drawn from the continuous uniform distribution $\mathcal{U}(0, L)$ (1995). This can be described as follows:

Let $X_1, X_2, \ldots, X_{2k+1}$ be the order statistics of $\mathcal{U}(0, L)$, such that $f_X(x) = \frac{1}{L}$. Then their joint probability density function is given by:

$$\begin{aligned} f_{X_1} \ldots f_{X_{2k+1}} (X_1, \ldots, X_{2k+1}) &= \begin{cases} (2k+1)! \prod_{i=1}^{2k+1} f_X(X_i) & \text{for } -\infty < X_1 < X_2 < \ldots < X_{2k+1} < \infty \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} \frac{(2k+1)!}{L^{(2k+1)}} & \text{for } -\infty < X_1 < X_2 < \ldots < X_{2k+1} < \infty \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Then for $s_1 = X_2, s_2 = X_4, \ldots, s_k = X_{2k}$, their joint distribution is:

$$
\begin{aligned}
f(s_1, \ldots, s_k) &= f(X_2, \ldots, X_{2k}) \\
&= \int_0^{X_2} \int_{X_2}^{X_4} \cdots \int_{X_{2k}}^{L} \frac{(2k+1)!}{L^{2k+1}} dX_1 dX_3 \ldots dX_{2k+1} \\
&= \frac{(2k+1)!}{L^{2k+1}} (X_2 - 0)(X_4 - X_2) \ldots (L - X_{2k}) \\
&= \frac{(2k+1)!}{L^{2k+1}} \prod_{j=1}^{k+1} (s_j - s_{j-1}) \qquad (7)
\end{aligned}
$$

3.4 Step types

The details for each step in the Reversible Jump MCMC algorithm are described below. The first two step types, a change of height and a change of position, are fairly straightforward and are based on Metropolis Hastings, while the remaining two steps, birth and death, change the dimension of the model - a condition which cannot be handled by Metropolis Hastings alone.

3.5 MH: Changing Heights

The acceptance probability for each type of transition is found by following the familiar MH algorithm structure: min (1, likelihood ratio × prior ratio × proposal ratio).

Under a change of height step, a height is randomly selected, $h_j, j \sim \mathcal{U}(1, \ldots, k+1)$, and a change is proposed, h'_j, with the relation:

$$
\log\left(\frac{h'_j}{h_j}\right) \sim \mathcal{U}(-a, a), \qquad (8)
$$

where we chose $a = 0.1$.

The value of a is uninformed, but was tuned to $a = 0.1$ after conducting several pilot runs. The acceptance probability can be derived as follows:

As defined previously in Equation 6, the prior distribution for heights leads to the following prior ratio:

$$
\begin{aligned}
\frac{p(h')}{p(h)} &= \frac{\beta^\alpha \, {h'_j}^\alpha \, e^{-\beta h'_j}}{\Gamma(\alpha) \, h'_j} \div \frac{\beta^\alpha \, h_j^\alpha \, e^{-\beta h_j}}{\Gamma(\alpha) \, h_j} \\
&= \frac{{h'_j}^\alpha \, h_j \, e^{-\beta h'_j}}{h'_j \, h_j^\alpha \, e^{-\beta h_j}} \\
&= {h'_j}^{\alpha-1} \, h_j^{1-\alpha} \, e^{-\beta(h'_j - h_j)} \\
&= \left(\frac{h'_j}{h_j}\right)^\alpha \frac{h_j}{h'_j} \, e^{-\beta(h'_j - h_j)} \qquad (9)
\end{aligned}
$$

The last line of (9) is left in this form for the purpose of matching Green's publication (1995). Then from (8), the proposal density is found:

$$u = \log h'_j - \log h_j \tag{10}$$

Then from Equation 10:

$$\begin{aligned}\frac{du}{dh'_j} &= \frac{1}{h_j} \cdot \frac{h_j}{h'_j} \\ &= \frac{1}{h'_j}\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{du}{dh_j} &= \frac{1}{h'_j} \cdot \frac{h'_j}{h_j} \\ &= \frac{1}{h_j}\end{aligned} \tag{12}$$

The proposal ratio is then:

$$\begin{aligned}\frac{p(h_j|h'_j)}{p(h'_j|h_j)} &= \frac{du}{dh_j} \div \frac{du}{dh'_j} \\ &= \frac{1}{h_j} \div \frac{1}{h'_j} \\ &= \frac{h'_j}{h_j}\end{aligned} \tag{13}$$

As the acceptance probability is min (1, likelihood ratio $\times$ prior ratio $\times$ proposal ratio) as mentioned previously, the acceptance probability is found to be:

$$\begin{aligned}&= \min\left\{1,\ \left(\text{likelihood ratio } \times \left(\frac{h'_j}{h_j}\right)^{\alpha} \frac{h_j}{h'_j}\ \times e^{-\beta(h'_j - h_j)}\ \times\ \frac{h'_j}{h_j}\right)\right\} \\ &= \min\left\{1,\ \left(\text{likelihood ratio } \times \left(\frac{h'_j}{h_j}\right)^{\alpha} \times e^{-\beta(h'_j - h_j)}\right)\right\}\end{aligned} \tag{14}$$

3.6 MH: Changing Locations

The acceptance probability for a change of position step is found in a similar way. An interval, j, is selected at random, $j \in \{1, \ldots, k\}$, which proposes a candidate breakpoint to be changed, denoted s_j. The prior distribution for step locations under the current state was found to be:

$$p(s|\, k, L) = \frac{(2k+1)!}{L^{2k+1}}(s_1 - s_0) \cdots \boldsymbol{(s_j - s_{j-1})(s_{j+1} - s_j)} \cdots (L - s_k) \tag{15}$$

While the prior under the proposed state is:

$$p(s'|\, k, L) = \frac{(2k+1)!}{L^{2k+1}}(s_1 - s_0) \cdots \boldsymbol{(s'_j - s_{j-1})(s_{j+1} - s'_j)} \cdots (L - s_k) \tag{16}$$

The prior ratio is thus:

$$\begin{aligned}\frac{p(s')}{p(s)} &= \frac{(2k+1)!\, L^{2k+1}\, (s_1 - s_0)(s_2 - s_1) \cdots (s'_j - s_{j-1})(s_{j+1} - s'_j) \cdots (L - s_k)}{(2k+1)!\, L^{2k+1}\, (s_1 - s_0)(s_2 - s_1) \cdots (s_j - s_{j-1})(s_{j+1} - s_j) \cdots (L - s_k)} \\ &= \frac{(s'_j - s_{j-1})(s_{j+1} - s'_j)}{(s_j - s_{j-1})(s_{j+1} - s_j)} \end{aligned} \tag{17}$$

As with the change of height step, an interval, j, is selected at random. The new value of s_j is then proposed. The new value s'_j of the selected breakpoint is sampled from a truncated normal distribution with a mean of the selected breakpoint under the current state, s_j, and is bounded by the breakpoints on either side, that is:

$$s'_j \sim \mathcal{N}_{trunc.}\,(\mu = s_j,\, a = s_{j-1},\, b = s_{j+1},\, \sigma = \delta) \tag{18}$$

The value of δ is chosen to be 1, however, this choice is fairly arbitrary. As a way of improving the algorithm performance, it is considered that the value of δ could be tuned during the burn-in period, however this method was not utilised in this analysis. The decision to use a truncated normal distribution for the proposal ratio is a point of difference between this analysis and Green's (1995), and this decision came about due to poor mixing under Green's proposed uniform distribution. The probability density function of the proposal distribution is then:

$$f(s'_j; \mu = s_j,\, a = s_{j-1},\, b = s_{j+1},\, \sigma = \delta) = \frac{\phi\left(\dfrac{s'_j - s_j}{\delta}\right)}{\Phi\left(\dfrac{s_{j+1} - s_j}{\delta}\right) - \Phi\left(\dfrac{s_{j-1} - s_j}{\delta}\right)} \tag{19}$$

Where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function and cumulative distribution function of the standard normal distribution respectively (conditioned on $a \leq s'_j \leq b$).

The proposal ratio is expressed below, in Equation 20:

$$\frac{p(s|s')}{p(s'|s)} = \frac{\phi\left(\frac{s_j - s'_j}{\delta}\right)}{\Phi\left(\frac{s_{j+1} - s'_j}{\delta}\right) - \Phi\left(\frac{s_{j-1} - s'_j}{\delta}\right)} \div \frac{\phi\left(\frac{s'_j - s_j}{\delta}\right)}{\Phi\left(\frac{s_{j+1} - s_j}{\delta}\right) - \Phi\left(\frac{s_{j-1} - s_j}{\delta}\right)}$$

$$= \frac{\Phi\left(\frac{s_{j+1} - s_j}{\delta}\right) - \Phi\left(\frac{s_{j-1} - s_j}{\delta}\right)}{\Phi\left(\frac{s_{j+1} - s'_j}{\delta}\right) - \Phi\left(\frac{s_{j-1} - s'_j}{\delta}\right)} \tag{20}$$

Then, as before, the acceptance probability of a change of breakpoint step is:

$$= \min\left\{1,\ \left(\text{likelihood ratio } \times \frac{(s'_j - s_{j-1})(s_{j+1} - s'_j)}{(s_j - s_{j-1})(s_{j+1} - s_j)} \times \frac{f_{\text{trunc.}}(s_j|s'_j)}{f_{\text{trunc.}}(s'_j|s_j)}\right)\right\} \tag{21}$$

3.7 Birth Step

For a birth step, the details are slightly more complicated, but are described as follows:

We propose a new breakpoint, $s^\star$, sampled from the continuous uniform distribution between 0 and L. Trivially, it is required that $s^\star$ lies within exactly one interval, (s_j, s_{j+1}), with probability 1. If the proposed breakpoint is accepted, the number of breakpoints, k, is increased by one, and the breakpoints and heights are relabeled to adjust for the extra step. For example, if $s^\star$ is proposed, and it lies within the interval (s_j, s_{j+1}), then the indices of the breakpoints in the positive direction are increased by one. That is:

$$\begin{aligned}\text{Instead of } &(s_0, \cdots, s_j, s_{j+1}, \cdots, s_k)\\ \text{we now have } &(s_0, \cdots, s_j, s^\star, s_{j+1}, \cdots, s_k)\end{aligned}$$

Under the acceptance of the birth step, the number of breakpoints, k, increases by one, which also leads to an increase in dimension in both the heights and step locations. The addition of $s^\star$ requires the definition of the relationship between the two new heights and the original height, that is the perturbation of h_j to become h'_j and h'_{j+1}. This balance is found by way of a geometric mean, as prescribed by Green (1995). This is seen below in Equations 22 and 23:

$$(s^\star - s_j)\log(h'_j) \;+\; (s_{j+1} - s^\star)\log(h'_{j+1}) \;=\; (s_{j+1} - s_j)\log(h_j) \tag{22}$$

$$\frac{h'_{j+1}}{h'_j} = \frac{1-u}{u} \tag{23}$$

Where u is drawn uniformly from $[0, 1]$

In considering the acceptance probability for a birth step, under the requirement of detailed balance, it is also necessary to define the death step. For this case, it is given by the removal of the added breakpoint, and the subsequent height over the remaining interval. The new height, h'_j, over the interval, $(s'_j, s'_{j+1}) = (s_j, s_{j+2})$ is the weighted geometric mean:

$$(s_{j+1} - s_j)\log(h_j) \;+\; (s_{j+2} - s_{j+1})\log(h_{j+1}) \;=\; (s'_{j+1} - s'_j)\log(h'_j) \tag{24}$$

The acceptance probability for this step is calculated using an additional term which accounts for the change in dimension. The Jacobian matrix in this context is the Jacobian determinant of the parameter vector:

$$\mathbf{J_b} = \begin{vmatrix} \dfrac{\partial h'_j}{\partial h_j} & \dfrac{\partial h'_j}{\partial u} \\ \dfrac{\partial h'_{j+1}}{\partial h_j} & \dfrac{\partial h'_{j+1}}{\partial u} \end{vmatrix} \tag{25}$$

With the inclusion of this term, the acceptance probability, as stated by Green, is given by (1995):

$$\min\{1, \text{(likelihood ratio} \times \text{prior ratio} \times \text{proposal ratio} \times |\text{Jacobian}|)\} \tag{26}$$

The prior ratio is more complex, but is found in a similar manner as in the change of location or change of height steps defined previously. The prior for s under the current state is the same as was found in Equation 15 given by:

$$p(s|\,k,L) = \frac{(2k+1)!}{L^{2k+1}}(s_1 - s_0)\cdots\boldsymbol{(s_{j+1} - s_j)}\cdots(L - s_k) \tag{27}$$

While under the proposed state the prior is:

$$p(s'|\,k+1,L) = \frac{(2k+3)!}{L^{2k+3}}(s_1 - s_0)\cdots\boldsymbol{(s^\star - s_j)(s'_{j+1} - s^\star)}\cdots(L - s_k) \tag{28}$$

Then the prior ratio for the locations is given by:

$$\begin{aligned}
\frac{p(s'|\,k+1,L)}{p(s|\,k,L)} &= \frac{(2k+3)!\,L^{2k+1}\,(s_1 - s_0)\cdots(s^\star - s_j)(s'_{j+1} - s^\star)\cdots(L - s_k)}{(2k+1)!\,L^{2k+3}\,(s_1 - s_0)\cdots(s_{j+1} - s_j)\cdots(L - s_k)} \\
&= \frac{(2k+3)(2k+2)(s^\star - s_j)(s'_{j+1} - s^\star)}{L^2\,\left(s_{j+1} - s_j\right)}
\end{aligned} \tag{29}$$

The prior for h under the current state is the same as was found in Equation 6 given by:

$$p(h) = \frac{\beta^\alpha h_j^\alpha e^{-\beta h_j}}{\Gamma(\alpha) h_j}$$

While under the proposed state the prior is:

$$p(h') = \frac{\beta^\alpha h_j'^\alpha e^{-\beta h'_j}}{\Gamma(\alpha) h'_j} \cdot \frac{\beta^\alpha h_{j+1}'^\alpha e^{-\beta h'_{j+1}}}{\Gamma(\alpha) h'_{j+1}} \tag{30}$$

Then the prior ratio for the heights is given by:

$$\begin{aligned}
\frac{p(h')}{p(h)} &= \frac{\beta^\alpha h_j'^\alpha e^{-\beta h'_j} \beta^\alpha h_{j+1}'^\alpha e^{-\beta h'_{j+1}}}{\beta^\alpha h_j^\alpha e^{-\beta h_j}} \cdot \frac{\Gamma(\alpha) h_j}{\Gamma(\alpha) h'_j \Gamma(\alpha) h'_{j+1}} \\
&= \frac{\beta^\alpha h_j'^{(\alpha-1)} h_{j+1}'^{(\alpha-1)} e^{-\beta(h'_j + h'_{j+1})}}{\Gamma(\alpha) h_j^{(\alpha-1)} e^{-\beta h_j}} \\
&= \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{h'_j h'_{j+1}}{h_j}\right)^{\alpha-1} e^{-\beta(h'_j + h'_{j+1} - h_j)}
\end{aligned} \tag{31}$$

The prior ratio for the increase in number of breakpoints is simply:

$$\frac{p(k')}{p(k)} = \frac{p(k+1)}{p(k)} \tag{32}$$

Overall, by (29), (31), and (32), the prior ratio for the birth step is:

$$\frac{p(k',s',h')}{p(k,s,h)} = \frac{p(k+1)}{p(k)} \frac{2(k+1)(2k+3)}{L^2} \frac{(s^\star - s_j)(s_{j+1} - s^\star)}{(s_{j+1} - s_j)} \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{h'_j h'_{j+1}}{h_j}\right)^{\alpha-1} e^{-\beta(h'_j + h'_{j+1} - h_j)} \tag{33}$$

For the proposal ratio, the probability of choosing a birth step, b_k, or death step, d_{k+1}, is reliant on the value of k. The proposal ratio is given below, in Equation 34:

$$
\begin{aligned}
\frac{p(k|k')}{p(k'|k)} &= d_{k+1}\left(\frac{1}{k+1}\right) \div b_k\left(\frac{1}{L}\right) \\
&= \frac{d_{k+1}L}{b_k(k+1)}
\end{aligned} \tag{34}
$$

The final term in the acceptance probability is the Jacobian determinant, the matrix of all first order partial derivatives of the variables in the model space. The Jacobian term arises from the mapping of the parameter space under one condition to the next, and accounts for the change in dimension that occurs when a birth or death step happens. In a reversible-jump setting, the Jacobian matrix is largely sparse, as the parameters are mostly independent from one another and these sparse entries are not listed for readability. The resulting Jacobian for a birth step is given by:

$$
\mathbf{J_b} = \begin{vmatrix} \dfrac{\partial h'_j}{\partial h_j} & \dfrac{\partial h'_j}{\partial u} \\ \dfrac{\partial h'_{j+1}}{\partial h_j} & \dfrac{\partial h'_{j+1}}{\partial u} \end{vmatrix} \tag{35}
$$

Each element of Equation 35 is derived below, and in determining these elements, it is advantageous to restate the relations given previously in Equations 22 and 23:

$$
(s^\star - s_j)\log(h'_j) \;+\; (s_{j+1} - s^\star)\log(h'_{j+1}) \;=\; (s_{j+1} - s_j)\log(h_j) \tag{36}
$$

$$
\log(h'_{j+1}) - \log(h'_j) = \log\left(\frac{1-u}{u}\right) \tag{37}
$$

It is useful to rewrite 37 in the following forms:

$$
\log(h'_j) = \log(h'_{j+1}) + \log\left(\frac{u}{1-u}\right) \tag{38a}
$$

$$
\log(h'_{j+1}) = \log\left(\frac{1-u}{u}\right) + \log(h'_j) \tag{38b}
$$

By substituting (38b) into (36):

$$
\begin{aligned}
(s^\star - s_j)\log(h'_j) + (s_{j+1} - s^\star)\left[\log(h'_j) + \log\left(\frac{1-u}{u}\right)\right] &= (s_{j+1} - s_j)\log(h_j) \\
(s_{j+1} - s_j)\log(h'_j) + (s_{j+1} - s^\star)\log\left(\frac{1-u}{u}\right) &= (s_{j+1} - s_j)\log(h_j)
\end{aligned} \tag{39}
$$

Then by implicit differentiation with respect to h_j:

$$
\begin{aligned}
(s_{j+1} - s_j)\frac{1}{h'_j}\frac{\partial h'_j}{\partial h_j} &= (s_{j+1} - s_j)\frac{1}{h_j} \\
\frac{\partial h'_j}{\partial h_j} &= \frac{h'_j}{h_j}
\end{aligned} \tag{40}
$$

In the same way, by substituting (38a) into (36):

$$(s^\star - s_j)\left[\log(h'_{j+1}) - \log\left(\frac{1-u}{u}\right)\right] + (s_{j+1} - s^\star)\log(h'_{j+1}) = (s_{j+1} - s_j)\log(h_j)$$
$$(s_{j+1} - s_j)\log(h'_{j+1}) - (s^\star - s_j)\log\left(\frac{1-u}{u}\right) = (s_{j+1} - s_j)\log(h_j)$$

And again by implicit differentiation with respect to h_j:

$$(s_{j+1} - s_j)\frac{1}{h'_{j+1}}\frac{\partial h'_{j+1}}{\partial h_j} = (s_{j+1} - s_j)\frac{1}{h_j}$$
$$\frac{\partial h'_{j+1}}{\partial h_j} = \frac{h'_{j+1}}{h_j} \tag{41}$$

Then for the other two terms, a similar approach is taken:

By substituting (38a) into (36) and rearranging:

$$\log(h'_{j+1}) + \left(\frac{s_{j+1} - s^\star}{s^\star - s_j}\right)\log(h'_{j+1}) = \log\left(\frac{1-u}{u}\right) + \left(\frac{s_{j+1} - s_j}{s^\star - s_j}\right)\log(h_j)$$
$$(s_{j+1} - s_j)\log(h'_{j+1}) = (s^\star - s_j)\log\left(\frac{1-u}{u}\right) + (s_{j+1} - s_j)\log(h_j) \tag{42}$$

And then differentiating (42) with respect to u:

$$\frac{(s_{j+1} - s_j)}{h'_{j+1}}\frac{\partial h'_{j+1}}{\partial u} = \frac{(s^\star - s_j)}{(u-1)u}$$
$$\frac{\partial h'_{j+1}}{\partial u} = h'_{j+1}\frac{(s^\star - s_j)}{(s_{j+1} - s_j)}\frac{1}{(u-1)u} \tag{43}$$

Finally, the last term is found:

$$(s_{j+1} - s_j)\log(h'_j) = (s_{j+1} - s_j)\log(h_j) + (s_{j+1} - s^\star)\log\left(\frac{u}{1-u}\right) \tag{44}$$

And taking the differential with respect to u yields:

$$\frac{(s_{j+1} - s_j)}{h'_j}\frac{\partial h'_j}{\partial u} = \frac{(s^\star - s_j)}{u(1-u)}$$
$$\frac{\partial h'_j}{\partial u} = h'_j\frac{(s_{j+1} - s^\star)}{(s_{j+1} - s_j)}\frac{1}{u(1-u)} \tag{45}$$

When each term is substituted into (35):

$$
\begin{aligned}
\mathbf{J_b} &= \left| \frac{\partial h'_j}{\partial h_j} \cdot \frac{\partial h'_{j+1}}{\partial u} - \frac{\partial h'_j}{\partial u} \cdot \frac{\partial h'_{j+1}}{\partial h_j} \right| \\
&= \left| \left[\frac{h'_j}{h_j} h'_{j+1} \frac{(s^\star - s_j)}{(s_{j+1} - s_j)} \frac{1}{(u-1)u} \right] - \left[\frac{h'_{j+1}}{h_j} h'_j \frac{(s_{j+1} - s^\star)}{(s_{j+1} - s_j)} \frac{1}{u(1-u)} \right] \right| \\
&= \left| \left(\frac{h'_j h'_{j+1}}{h_j(s_{j+1} - s_j)} \right) \left(\frac{(s^\star - s_j) + (s_{j+1} - s^\star)}{u(u-1)} \right) \right| \\
&= \left| \left(\frac{h'_j h'_{j+1}}{h_j} \right) \left(\frac{1}{u(u-1)} \right) \right| \qquad (46)
\end{aligned}
$$

This is then solved by substituting u into (46) where:

$$
u = \frac{h'_j}{h'_j + h'_{j+1}} \qquad (47)
$$

$$
\mathbf{J_b} = \left| - \left(\frac{h'_j h'_{j+1}}{h_j} \right) \left(\frac{(h'_j + h'_{j+1})^2}{h'_j h'_{j+1}} \right) \right|
$$

$$
\mathbf{J_b} = \frac{(h'_j + h'_{j+1})^2}{h_j} \qquad (48)
$$

With all of the combined ratios from (33), (34), and (48), the acceptance probability in the form of $\min\left(1,\ \text{likelihood ratio} \times \text{prior ratio}\ \times\ \text{proposal ratio} \times |\ \text{Jacobian}|\right)$ can then be constructed.

3.8 Death Step

For a Death step, one breakpoint, s_{j+1}, is sampled randomly and becomes the candidate for removal. As detailed above in Equation 24, under the acceptance of the step, the new height over the interval $(s'_j, s'_{j+1}) = (s_j, s_{j+2})$ is denoted h'_j. The prior distribution of s under the current state is:

$$p(s|\, k, L) = \frac{(2k+1)!}{L^{2k+1}}(s_1 - s_0)\cdots \boldsymbol{(s_j - s_{j-1})(s_{j+1} - s_j)}\cdots(L - s_k) \tag{49}$$

While under the proposed state the prior is

$$p(s'|\, k-1, L) = \frac{(2k-1)!}{L^{2k-1}}(s_1 - s_0)\cdots \boldsymbol{(s_{j+1} - s_{j-1})}\cdots(L - s_k) \tag{50}$$

Then the prior ratio for breakpoints is:

$$\begin{aligned}\frac{p(s'|\, k-1, L)}{p(s|\, k, L)} &= \frac{(2k-1)!\, L^{2k-1}\,(s_1 - s_0)\cdots(s_{j+1} - s_{j-1})\cdots(L - s_k)}{(2k+1)!\, L^{2k+1}\,(s_1 - s_0)\cdots(s_j - s_{j-1})(s_{j+1} - s_j)\cdots(L - s_k)} \\ &= \frac{L^2(s_{j+1} - s_{j-1})}{(2k+1)(2k)(s_j - s_{j-1})(s_{j+1} - s_j)}\end{aligned} \tag{51}$$

The prior distribution of h under the current state is:

$$p(h) = \frac{\beta^\alpha h_j^\alpha e^{-\beta h_j}}{\Gamma(\alpha) h_j} \cdot \frac{\beta^\alpha h_{j+1}^\alpha e^{-\beta h_{j+1}}}{\Gamma(\alpha) h_{j+1}} \tag{52}$$

While under the proposed state the prior is

$$p(h') = \frac{\beta^\alpha h_j'^\alpha e^{-\beta h'_j}}{\Gamma(\alpha) h'_j} \tag{53}$$

Then, as before, the prior ratio for heights is:

$$\begin{aligned}\frac{p(h')}{p(h)} &= \frac{\beta^\alpha h_j'^\alpha h_j h_{j+1} e^{-\beta h'_j}}{\beta^{2\alpha} h'_j h_j^\alpha h_{j+1}^\alpha e^{-\beta h_j} e^{-\beta h_{j+1}}} \cdot \frac{\Gamma(\alpha)^2}{\Gamma(\alpha)} \\ &= \frac{h_j^{(\alpha-1)} e^{-\beta h'_j}}{\beta^\alpha (h_j h_{j+1})^{(\alpha-1)} e^{-\beta(h_j + h_{j+1})}} \cdot \Gamma(\alpha) \\ &= \frac{\Gamma(\alpha)}{\beta^\alpha}\left(\frac{h'_j}{h_j h_{j+1}}\right)^{\alpha-1} e^{-\beta(h'_j - h_j - h_{j+1})}\end{aligned} \tag{54}$$

In the case of a successful death step, the number of breakpoints, k, decreases by one, and also leads to a change in the prior for the number of breakpoints. The prior ratio describing this change is:

$$\frac{p(k')}{p(k)} = \frac{p(k-1)}{p(k)} \tag{55}$$

Overall, by (51), (54), and (55):

$$\frac{p(k',s',h')}{p(k,s,h)} = \frac{p(k-1)}{p(k)} \frac{L^2}{(2k+1)(2k)} \frac{\left(s'_{j+1}-s'_j\right)}{\left(s_{j+1}-s_j\right)\left(s_{j+2}-s_{j+1}\right)} \frac{\Gamma(\alpha)}{\beta^\alpha} \left(\frac{h'_j}{h_j h_{j+1}}\right)^{\alpha-1} e^{-\beta(h'_j-h_j-h_{j+1})} \tag{56}$$

The proposal ratio is given below in Equation 57:

$$\begin{aligned}\frac{p(k|k')}{p(k'|k)} &= b_{k-1}\left(\frac{1}{L}\right) \div d_k \left(\frac{1}{k}\right)\\ &= \frac{b_{k-1}\,k}{d_k\,L}\end{aligned} \tag{57}$$

Under the acceptance of the death step, the number of breakpoints, k, decreases by one, which leads to a decrease in the dimension of heights and break points respectively. The Jacobian related to this change in the dimension of the parameter space of the model is expressed as:

$$\mathbf{J_d} = \begin{vmatrix} \frac{\partial h'_j}{\partial h_j} & \frac{\partial h'_j}{\partial h_{j+1}} \\ \frac{\partial u'}{\partial h_j} & \frac{\partial u'}{\partial h_{j+1}} \end{vmatrix} \tag{58}$$

Each term is found in a similar way as the birth step, with the weighted geometric mean found in Equation 24 being used to solve the system of equations along with the perturbation given by Equation 23. It is simpler to express (23) as:

$$u' = \frac{h_j}{h_j + h_{j+1}} \tag{59a}$$

$$h_{j+1} = h_j\left(\frac{1-u'}{u'}\right) \tag{59b}$$

$$h_j = h_{j+1}\left(\frac{u'}{1-u'}\right) \tag{59c}$$

By substituting (59c) into (24):

Restating (24):

$$(s'_{j+1}-s'_j)\log(h'_j) \;=\; (s_{j+1}-s_j)\log(h_j) \;+\; (s_{j+2}-s_{j+1})\log(h_{j+1})$$

Then by taking the partial derivative with respect to h_j:

$$\begin{aligned}\frac{(s_{j+2}-s_j)}{h'_j}\frac{\partial h'_j}{\partial h_j} &= \frac{(s_{j+1}-s_j)}{h_j}\\ \frac{\partial h'_j}{\partial h_j} &= \left(\frac{s_{j+1}-s_j}{s_{j+2}-s_j}\right)\frac{h'_j}{h_j}\end{aligned} \tag{60}$$

And by taking the partial derivative of (24) with respect to h_{j+1}:

$$\begin{aligned}\frac{(s_{j+2}-s_j)}{h'_j}\frac{\partial h'_j}{\partial h_{j+1}} &= \frac{(s_{j+2}-s_{j+1})}{h_{j+1}}\\ \frac{\partial h'_j}{\partial h_{j+1}} &= \left(\frac{s_{j+2}-s_{j+1}}{s_{j+2}-s_j}\right)\frac{h'_j}{h_{j+1}}\end{aligned} \tag{61}$$

The remaining terms are found by differentiating (59a), which is expressed as Equation 62 below:

$$\log(1-u') - \log u' = \log h_{j+1} - \log h_j \tag{62}$$

By taking the partial differential of (62) with respect to h_j:

$$\begin{aligned} \left(-\frac{1}{1-u'} - \frac{1}{u'}\right)\frac{\partial u'}{\partial h_j} &= -\frac{1}{h_j} \\ \frac{\partial u'}{\partial h_j} &= -\frac{(u'-1)u'}{h_j} \end{aligned} \tag{63}$$

And with respect to h_{j+1}:

$$\begin{aligned} \left(-\frac{1}{1-u'} - \frac{1}{u'}\right)\frac{\partial u'}{\partial h_{j+1}} &= \frac{1}{h_{j+1}} \\ \frac{\partial u'}{\partial h_{j+1}} &= \frac{(u'-1)u'}{h_{j+1}} \end{aligned} \tag{64}$$

$$\tag{65}$$

When each term is substituted into (58):

$$\mathbf{J_d} = \left|\left(\frac{\partial h'_j}{\partial h_j}\cdot\frac{\partial u'}{\partial h_{j+1}}\right) - \left(\frac{\partial h'_j}{\partial h_{j+1}}\cdot\frac{\partial u'}{\partial h_j}\right)\right|$$

$$= \left|\left(\frac{(s_{j+1}-s_j)}{(s_{j+2}-s_j)}\frac{h'_j}{h_j}\frac{(u'-1)u'}{h_{j+1}}\right) - \left(\frac{(s_{j+2}-s_{j+1})}{(s_{j+2}-s_j)}\frac{h'_j}{h_{j+1}}\left(-\frac{(u'-1)u'}{h_j}\right)\right)\right|$$

And by (59a):

$$= \left|-\left(\frac{(s_{j+1}-s_j)}{(s_{j+2}-s_j)}\frac{h'_j}{(h_j+h_{j+1})^2}\right) - \left(\frac{(s_{2+1}-s_{j+1})}{(s_{j+2}-s_j)}\frac{h'_j}{(h_j+h_{j+1})^2}\right)\right|$$

$$\mathbf{J_d} = \frac{h'_j}{(h_j+h_{j+1})^2} \tag{66}$$

With all of the combined ratios from (56), (57), and (66), the acceptance probability in the form of min (1, likelihood ratio $\times$ prior ratio $\times$ proposal ratio $\times$ Jacobian) can then be constructed. The process outlined above for finding the Jacobian determinant under a Death Step was not required, as $\mathbf{J_d} = \mathbf{J_b}^{-1}$. The requirement of dimension matching is fulfilled, which is what makes the algorithm reversible.

3.9 Model Selection Criteria

One commonly used way to perform model selection in Bayesian Statistics is by evaluating the Deviance Information Criterion (DIC) for each model in turn, and selecting the model with the lowest DIC.

The Deviance Information Criterion (DIC) is calculated in the conventional manner (Spiegelhalter et al. 2002):

$$
\begin{aligned}
D\left(\theta\right) &= -2\log\left(p\left(y|\theta\right)\right) + C \\
p_D &= \bar{D} - D\left(\bar{\theta}\right) \\
DIC &= D\left(\bar{\theta}\right) + 2p_D
\end{aligned}
$$

4 Results

Estimates based on both the MH and RjMCMC models were found successfully for each category of Crime and Punishment, with posterior estimates of k under RjMCMC of k ranging from 3 to 43 breakpoints. Overall, the estimates found by the RjMCMC methods were not incredibly dissimilar from those found by the MH algorithm, however the number of iterations and the algorithm run time was drastically reduced under RjMCMC.

4.1 Algorithm Performance

For the Metropolis Hastings modelling, a number of pilot runs were undertaken, and the following settings were found to be adequate for analysis. For each category of crime and punishment, a single chain was constructed and estimates were calculated at the end of 50,000 iterations. A burn-in period of 20,000 iterations was established, in order to improve model convergence and performance. This burn-in period became more important as the value of k increased, as a larger number of intervals were being sampled from at each iteration, so each interval or breakpoint was less likely to have been sampled when comparing to a model with lower k. The choice of 50,000 iterations, thinned to 500 samples after the first 20,000 were discarded comes from a balance of ensuring that the chain has stabilized, while being conservative in the amount of data that was being estimated and saved to disk at each iteration. As it was, when running the code for 50,000 iterations per k, $k \in \{0, \dots, 60\}$, it took approximately 80 minutes for the algorithm to complete.

For the reversible jump MCMC, a similar process was undertaken, and the algorithm was run for each of the crime and punishment categories. The number of iterations, burn-in, and size of thinned sample was the same as for the Metropolis Hastings modelling, however, as the value of k was not fixed, the chain only had to be run once. The initial setting was $k = 3$, a choice that was made from convenience, as the algorithm is able to quickly increase the dimension of the model if it is necessary, so the target distribution is reached in sufficient time, regardless of the initial state. The code took between 16 and 17 mintues to run for each category of data, and as such was much more efficient in finding estimates.

A sample of the trace plots for different subsets of data and values of k was assessed visually, and it was determined that the process had converged.

The analysis was carried out using R version 3.5.1, on a workstation equipped with 16GB RAM and an Intel(R) Core(TM) i7-6700HQ, 2.60 GHz processor, and running the Windows 10 Home (version 1803) operating system (64-bit).

4.2 An Illustrative Example - Sexual Offences

In order to illustrate the difference in results under the Metropolis Hastings design, we look at one crime category, sexual offences, in greater detail. The data is seen below in Figure 1:

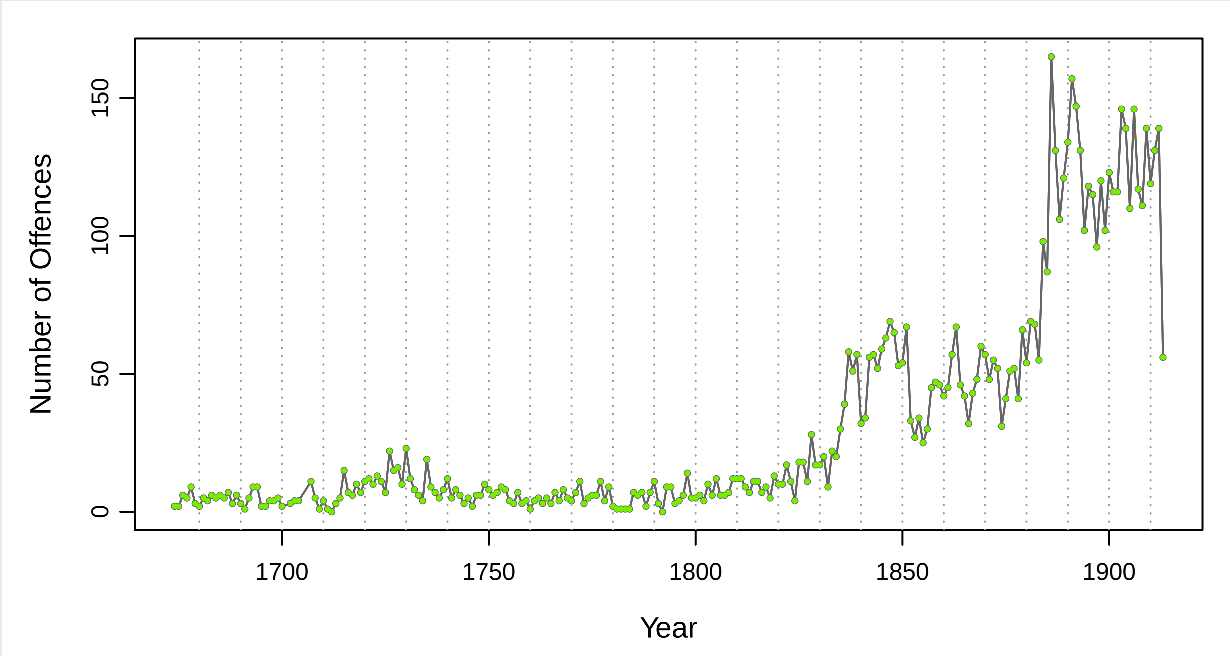


Figure 1: Number of Sexual Offence Crimes heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

From visual inspection, it is apparent that the rate of sexual offences was comparatively low until the 1830's, when the crime rate rose sharply, which was followed by a turbulent period of an elevated rate of offending. Again, in the 1880's, the number of sexual offences rose dramatically, with offending reaching its peak in 1886. The remaining years of records saw a comparatively higher incidence of sexual crimes, with a large variation seen from year to year.

A graph of the DIC for each value of k can be seen below, with the optimal value of k found under both the MH and RjMCMC algorithms highlighted as appropriate:

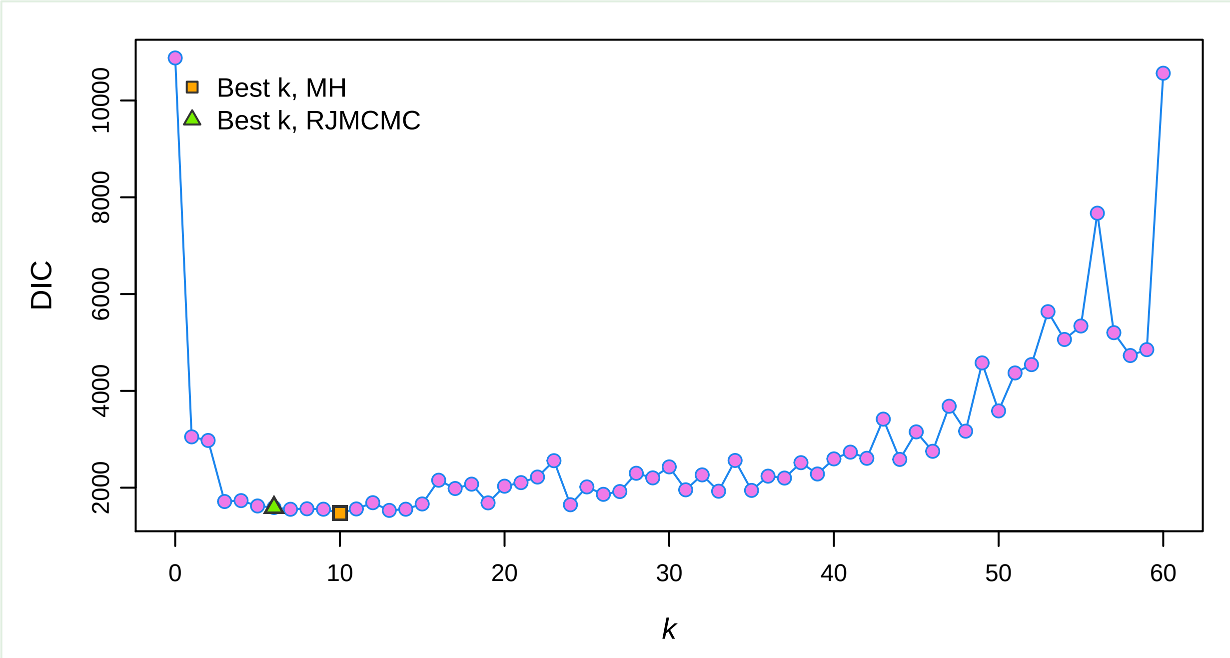


Figure 2: Plot of DIC per value of k when modelling the number of Sexual Offence Crimes heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

The plot in 2 shows that there is a large improvement in model fit with increasing k until $k = 10$, after which additional breakpoints do not seem to better explain the data. By using the DIC as the measure of model fit, the Metropolis Hastings algorithm found that 10 breakpoints was optimal for the Sexual Offences data, which is slightly larger than the posterior k found by the reversible jump algorithm, $k = 6$.

The estimates and related 95% Credible Intervals found for each year, conditional on the best model according to DIC, $k = 10$, can be seen below in Figure 3, with the related estimates for h and s shown in Tables 1 and 2 on the following pages:

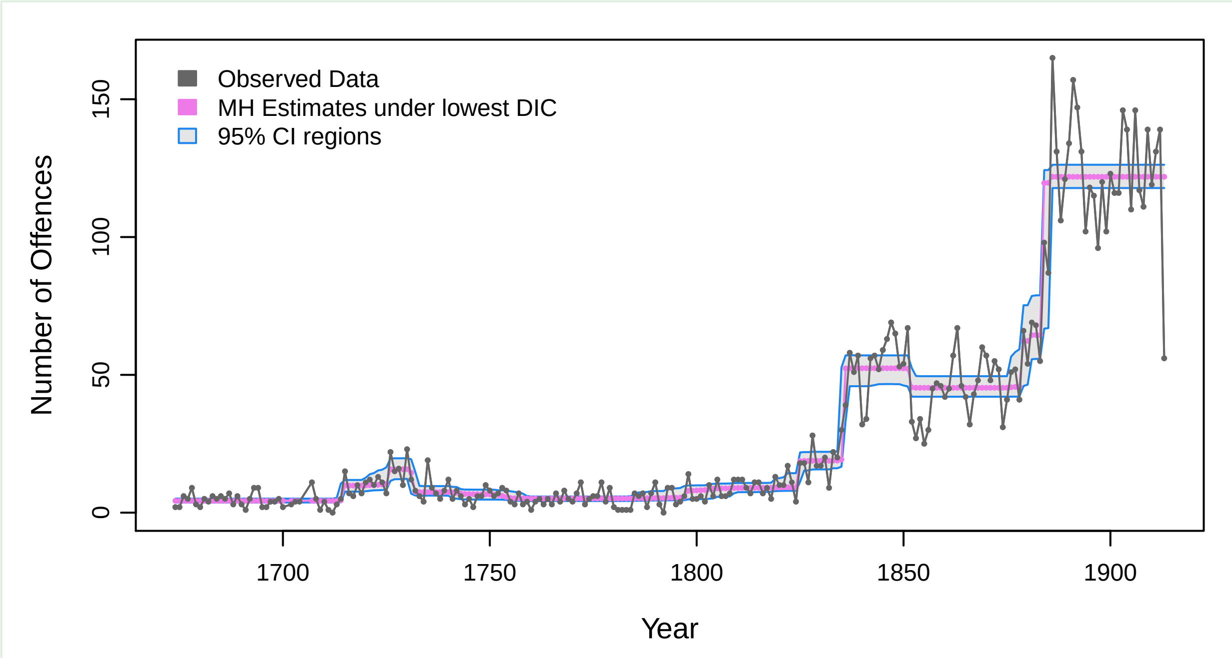


Figure 3: MH estimates from lowest DIC, Modelling the number of Sexual Offence Crimes heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

h	Posterior Estimate	95% CI
0	4.3	(3.7, 5.1)
1	9.8	(7.6, 11.9)
2	15.7	(12.3, 19.8)
3	7.4	(6.2, 9.6)
4	5.1	(4.2, 5.9)
5	8.7	(5.5, 10.5)
6	18.2	(8.7, 21.6)
7	52.4	(17.4, 57.1)
8	45.3	(42.1, 49.5)
9	64.4	(55.8, 78.8)
10	121.9	(117.8, 126.2)

Table 1: MH Posterior Estimates for h, conditioned on k = 10

s	Posterior Estimate	95% CI	Year
1	41.3	(40, 41.9)	1716
2	52.3	(46.8, 53.4)	1727
3	58.3	(57, 59.7)	1733
4	80.4	(67.3, 85.7)	1755
5	124	(111, 131.9)	1798
6	151.3	(129.2, 152)	1826
7	162.3	(151.2, 163)	1837
8	178.2	(161.5, 179.5)	1853
9	205.5	(202.7, 207.7)	1880
10	210.9	(210, 212.9)	1885

Table 2: MH Posterior Estimates for s, conditioned on k = 10

Next, we compare these estimates to the ones found by the RjMCMC method. After thinning, the algorithm found between six and ten breakpoints, with $k = 6$ being the posterior mode. The posterior estimates per year, conditioned on $k = 6$ are seen below in Figure 4, along with the related 95% Credible Intervals:

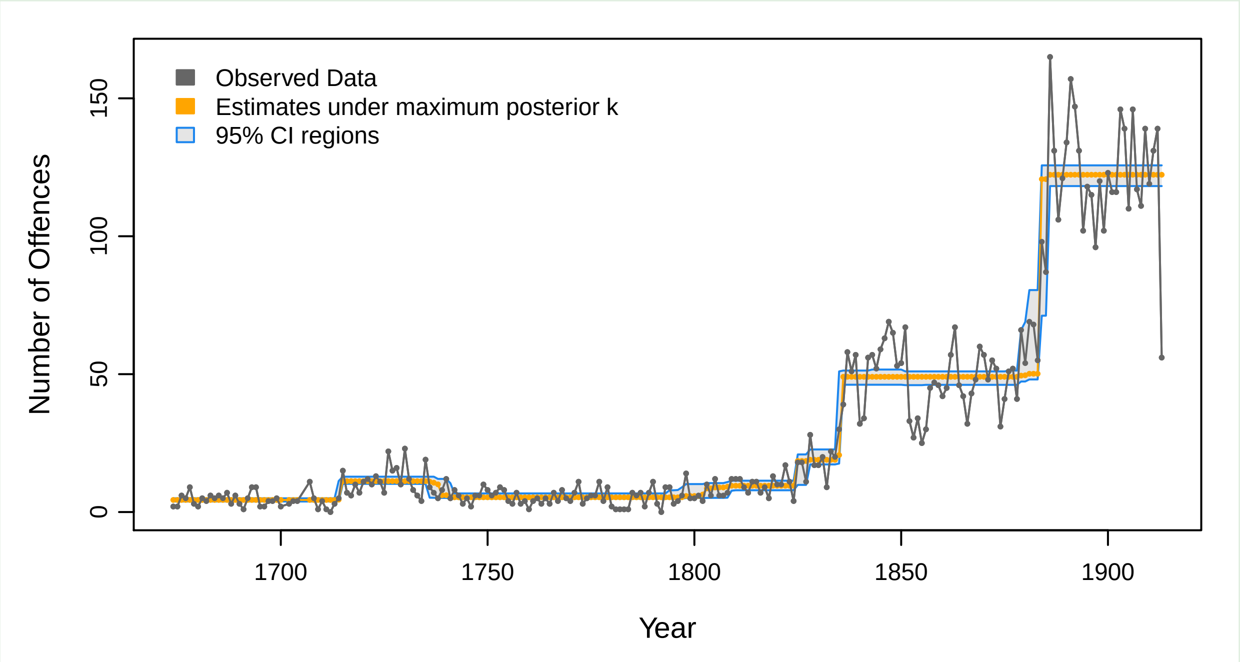


Figure 4: RjMCMC estimates conditional on k = 6 (posterior mode), Modelling the number of Sexual Offence Crimes heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

Conditioned on $k = 6$, we also found the following estimates and 95% CIs for breakpoints and intensities (Tables 3, 4):

h	Posterior Estimate	95% CI
0	4.3	(3.8, 4.8)
1	11.1	(10.1, 12.8)
2	5.5	(4.9, 6.8)
3	9.4	(7.9, 11)
4	18.8	(16.8, 22.6)
5	49.3	(47.3, 51)
6	121.3	(118.2, 127.1)

Table 3: Posterior Estimates for h, conditioned on k = 6

s	Posterior Estimate	95% CI	Year
1	41.4	(40.2, 42)	1716
2	67.3	(62.2, 69.6)	1742
3	130	(123.2, 135.9)	1804
4	151.5	(151.1, 154.5)	1826
5	162.4	(161.1, 163)	1837
6	210.5	(210.1, 211)	1885

Table 4: Posterior Estimates for s, conditioned on k = 6

Furthermore, the estimated posterior distribution for k, and the Model Averaged (MA) estimates per year can be seen below in Figure 5, Table 5, and Figure 6.

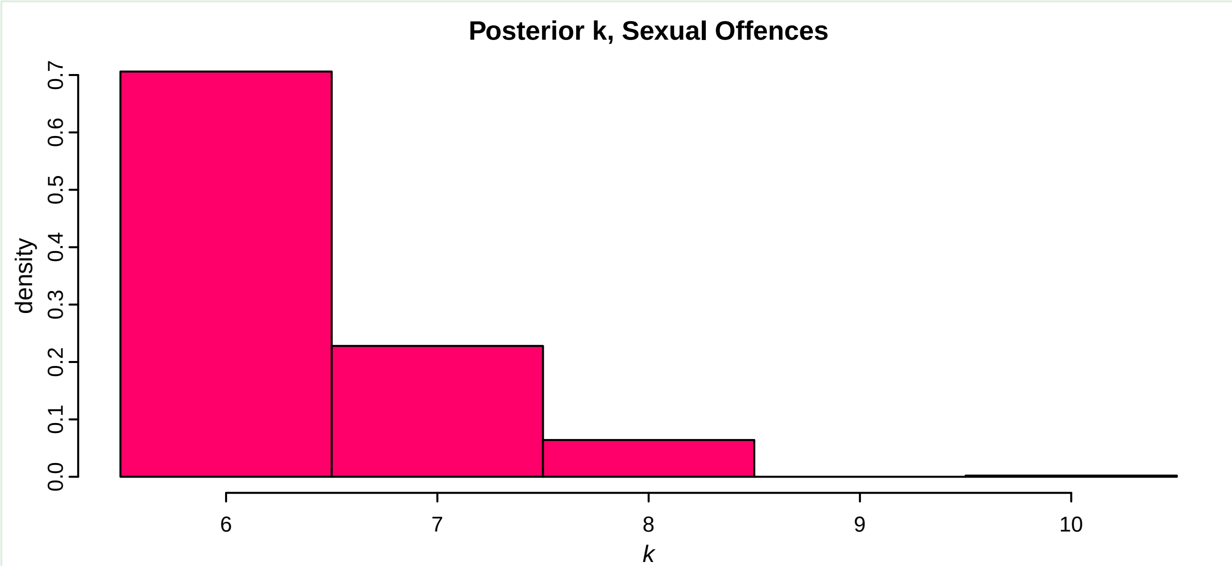


Figure 5: Estimated Posterior Distribution of k, Crimes by Offence, Sexual Offences (Old Bailey Online 2018a).

k	Proportion
6	0.706
7	0.228
8	0.064
10	0.002

Table 5: Posterior estimate for k

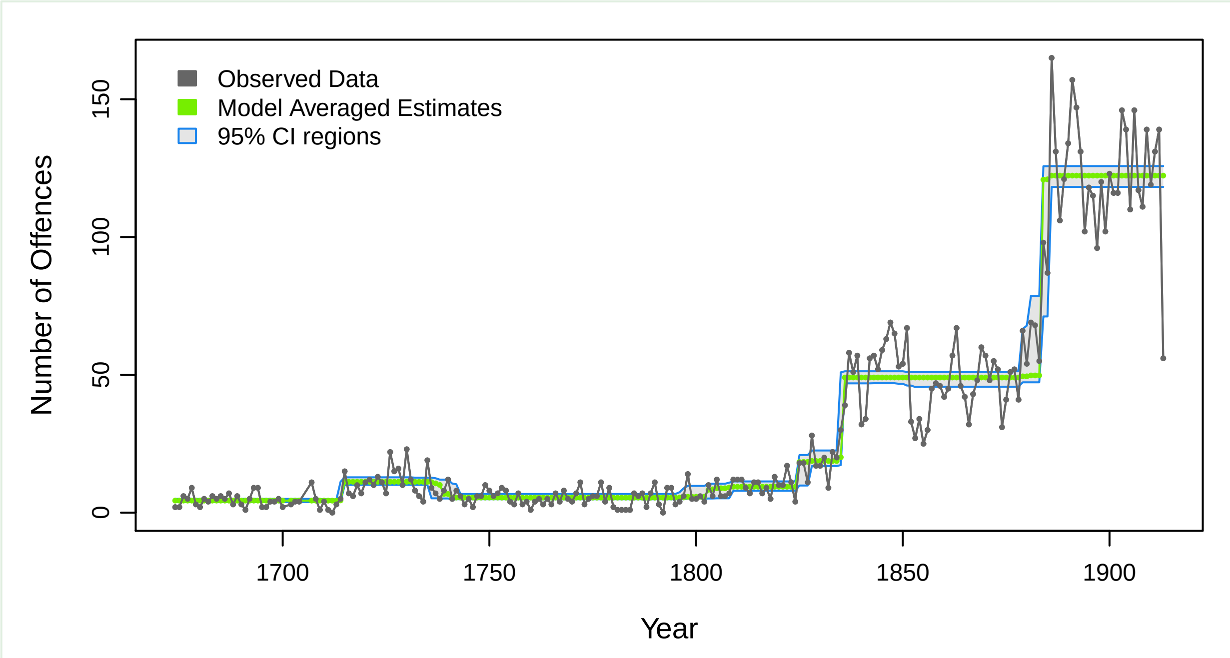


Figure 6: RjMCMC Model Averaged Estimates per Year, Crimes by Offence, Sexual Offences (Old Bailey Online 2018a).

There are subtle differences between the two models, however they have largely produced similar posterior estimates, and with comparable estimation error. Similar DIC plots have been produced for each category of crime or punishment, and are compared with the posterior estimates of k in the Supplementary Figures in the Appendix.

5 Discussion

Overall, the Reversible jump MCMC procedure performed well for this application, and a number of breakpoints can be related to historical and judicial events of note. A number of adaptations were made to Green's proposal, which allowed for these results to be found after coming across obstacles in the early stages of this analysis.

5.1 Data

The historical importance of the *Proceedings* is unquestionable according to historian John Langbein, who is quoted by the Old Bailey Online as suggesting that they are "probably the best accounts we shall ever have of what transpired in ordinary English criminal courts before the later eighteenth century" (2018d). However, the completeness of these records cannot be assured, due to a variety of reasons.

Originally, the publications had the same "flavour of the traditional forms" of literature, namely that the content focused on trials that would be "likely to attract an audience" (Beattie 2001a). As cited by Beattie, according to "Trials and Criminal Biographies" by Michael Harris, as "early as 1678 the court of aldermen stepped in to control and regulate the publication of reports on the Old Bailey sessions" (2001a). Over the course of a few years, the publications became far more substantial and frequent, and in the 1860s were considered an (almost) "complete record of all the cases that had been tried, revealing for the first time in a systematic way the numbers of men and women convicted and acquitted, and the range of punishments imposed on the guilty" (Beattie 2001a). The *Proceedings* grew to include more detailed accounts over time, for a variety of reasons (Shoemaker 2008). Notably in 1775, the outgoing lord mayor, John Wilkes, proposed that omissions of trial details in the publications were "universally complained of", and regulations were gradually introduced (Shoemaker 2008). In 1778, it was suggested that the *Proceedings* should "contain a true, fair, and perfect narrative of the whole evidence upon the trial of every prisoner, whether he or she shall be convicted or acquitted" (Shoemaker 2008). The motivations behind the publishers and note takers have been called into question, as explored by Shoemaker, particularly considering that the patrons of the *Proceedings* were largely "the men and women who were most likely to be the victims of the thefts dominating the Old Bailey's dockets" (2008). While every effort could be made to provide readers with a full transcript of every trial, limitations in the publication medium made this impossible, as even when the *Proceedings* grew in length, there was not enough room to include all of the possible information (Shoemaker 2008).

Although the *Proceedings* eventually became a regular periodical, for the length of time between 1699 and 1714, there appears to be a conspicuous lack of information (Emsley, Hitchcock, and Shoemaker 2018c). There is no consensus as to why there appears to only be editions for approximately one-third of the sessions that should have taken place in that time, not to mention that "there are three years for which no Proceedings survive: 1701, 1705, and 1706" (Emsley, Hitchcock, and Shoemaker 2018c). Additionally, it should be considered that the number of punishments that were recorded do not necessarily reflect the number that were actually carried out. Less than one fifth of death sentences were realized, with many convicts being pardoned through benefit of clergy, or by way of partial verdicts (Emsley, Hitchcock, and Shoemaker 2019a). Depending on the evidence provided, the burden of proof required for conviction, and whether or not "the court thought a 'crime wave' was in process", a defendant could be spared capital punishment, or hanged to be made an example of (Emsley, Hitchcock, and Shoemaker 2019a). Questions can be posed over the completeness of the *Proceedings* as a whole, however they have still informed the opinions of many historians on crime and punishment (Shoemaker 2008), and are increasingly referred to in modern times - thanks in part to the digitization of the transcripts.

The digitization process was primarily undertaken by The Higher Education Digitisation Service at the University of Hertfordshire, by scanning images of microfilms of the original *Proceedings* and *Ordinary's Accounts* and compressing them into GIF and JPEG formats so that they may be viewed electronically as a part of the Old Bailey Online project (Emsley, Hitchcock, and Shoemaker 2018b). The majority of the texts were transcribed using Optical Character Recognition software, however, some discrepancies had to be resolved manually. This is mostly due to the loss of clarity from the media, as the earliest originals are "often faded or suffer from ‘bleed through’ (where print on the other side of the page interferes with the text)”. This is somewhat to be expected due to the age and condition of the original text.

A number of changes were made to the database over time, some of which became apparent over time span of this project. The Old Bailey Online staff have arranged their website in such a way that the “version number” can be seen in the footer of each page, and furthermore, the changes that came with each new version of a particular feature were summarized on their “What’s new” and “What’s new archive” pages under the section titled “The Project” (Emsley, Hitchcock, and Shoemaker 2018b). Of particular concern are some discrepancies in the update notes. As an example, it is documented that an error was fixed in both the Februrary 2018 update (Version 8.0) and the March 2015 update (Version 7.2), which produced incorrect results in the search feature, as male defendants were tagged with the occupational title of “wife”. While this particular example was not of immediate concern in this thesis, it was noted that over the course of the project, the total number of defendants tried at the Old Bailey varied. Earlier in the project, it was recorded that 252,552 defendants were heard at the Old Bailey, while at a later date this was changed to 252,509, i.e. 43 defendants were unaccounted for. Both of these totals were obtained under the same website version, however, it is suspected that other changes were made to the database that were not logged - either that the website reverted to a previous version for a period of time and this was not publicized, or that a change was made to the coding of the aggregator. In the interest of consistency, once I found this error, I downloaded all of the datasets used for this project at one time, and performed analyses on the data as it was in Version 8.0 (Old Bailey Online 2018a). While these errors do not appear to immediately affect the outcome of the statistics tool, the credibility of the search tool as a whole may suffer under scrutiny.

One possible improvement could be made by fine-tuning the OCR algorithm with a greater training sample to improve data integrity, although starting this process from scratch would be resource intensive. While this could reduce coding errors, a more worthwhile use of time could be to follow up on the outcome of specific cases, by searching other literature for evidence of whether a punishment was actually carried out or not. Without a better way of estimating the number of penalties that were administered, the incidence of punishment may be heavily skewed, leading to incorrect conclusions about the crime rate at the time. It is this uncertainty that directed my focus more towards the data considered by crime offences for deeper consideration in this thesis.

It is worth noting that this thesis did not address the potential difficulties surounding the changing population of London over the time of the Old Bailey. While it is theoretically possible to obtain population estimates for the City of London or England in general, the Old Bailey Courthouse represents a particular section of criminals as only particularly serious crimes were heard, and depending on the severity of the offending, the potential population that the defendant could come from could vary geographically (Emsley, Hitchcock, and Shoemaker 2018a). Adding further uncertainty was the presence of wars and conflict, wherein it became difficult to determine whether the potential population of offenders or victims were to be considered.

5.2 Methods

In the process of initial analysis, it was found that the proposal distributions supplied by Green were insufficient for the model (1995). In particular, under a change of position step, we found that a uniform proposal distribution was too wide ranging, and that the algorithm would not converge. Using the narrower, more focused truncated normal proposal distribution allowed for the chain to converge. It is possible that this narrower proposal distribution could have required a longer time to reach the target posterior, however this was not an issue in this instance. If this were an obstacle in future studies, I would propose that the value of δ, the measure of spread, is used as a tuning parameter in the burn-in period.

In hindsight, the use of even-numbered order statistics as the prior distribution for breakpoint locations may not have been advantageous for this data. According to Green, this ensured that not too many short jumps were proposed, however, we encountered this problem regardless, and the use of a somewhat obscure prior could be off-puting to some readers.

5.3 Limitations of using DIC as a model selection criterion

The use of deviance information criterion (DIC) for model comparison under a Bayesian paradigm has been widely debated, with the criticisms, as summarized by Plummer, suggesting that one must face "a choice between hedging the use of DIC with a discussion of its potential limitations [...] or trusting the expert judgment that 'experience with DIC to date suggests that it works remarkably well' " (2008). In essence, DIC is seen as a "Bayesian analogue of classical model-choice criteria, such as the Akaike information criterion (AIC)", however, as with any opinion on the definitive use of a given model selection criterion, there is no clear consensus of exactly when and how it should be used. It has been noted that the justification for the use of DIC is only shown to be valid heuristically, and lacks the formality required. In particular, the choice of penalty term, p_D, can affect the overall model selection in a variety of situations, notably by under-penalizing models with greater complexity when the (effective) number of parameters $\boldsymbol{\theta}$ is "much smaller than" the number of observations. As suggested in Plummer's summary, there have been many alternatives put forward to remedy this possible source of obsfucation, but with the adendum that all of these adaptations were similarly unconvincing.

In this thesis, the use of DIC for model selection is included for the fixed k model, largely for the sake of comparison with the Reversible Jump results. The benefits of DIC are seen particularly in MCMC applications, and, according to Plummer, is so widely used that it has earned its inclusion in textbooks concerning Bayesian data analysis (2008).

In the scope of this thesis, the effective number of parameters was not likely to be excessively large, however this judgement is largely *a posteriori*. In addition, under this specific set of circumstances, it would not be completely unfeasible to run multiple chains under an MCMC procedure due to run-time complexity, however, it was not immediately clear that this would *necessarily* be the case. This is particularly true for deeper analysis of this data, such as considering the crimes and punishments as simultaneous Poisson processes.

5.4 Greater implications

By focusing on the crime category of Sexual Offences, we can draw some relationships between the breakpoints found and changes in criminality. In particular, conditioned on the posterior mode of k, $k = 6$, the breakpoint on L, 162.4 (161.1, 163), or in terms of years 1837 (1836, 1838) nearly coincides with the year that rape ceased being a capital offence, 1841. Following this, there is a sharp increase in sexual offences, seen in the change in estimated average intensity from 18.8 offences per year prior to 1837, to 49.3 in the decades following.

Also in the category of sexual offending are penetrative homosexual acts, which were punishable by death until 1861 (Emsley, Hitchcock, and Shoemaker 2020). It was not until the Labouchere Amendment of 1885 that the punishment for homosexual acts was reduced to two years' incarceration, from the far more severe prospect of life imprisonment. With the relative reduction in severity of punishment, however, was a lower threshold for proof in order to obtain a conviction, and a redefinition of the scope of "illegal sexual activity" to include the somewhat ambiguous "gross indecency" between men. Over the course of the Old Bailely trials a number of offences were redefined to include more modern modes of crime, or by diversifying the range of subcategories of offending to allow for different levels of penalty; the Labouchere Amendment is just one example.

While not all breakpoints found by these methods can be directly attributed to changes in society, it is widely noted that there was a "notable spasm" of homophobia in 1890 following the prolific Cleveland Street prosecutions, and the potential evidence of this moral sensitivity is clearly seen in the dramatic increase in incidence of sexual crime after the estimated breakpoint found in 1885 (Emsley, Hitchcock, and Shoemaker 2020). Adding fuel to the fire is the prosecutions of Oscar Wilde in 1895 at the height of sexual offending in the *Proceedings*.

5.5 Future Work

In the process of analyzing The Old Bailey data, it was considered that the punishments and crimes could be considered in tandem. Given more time, the possibility of modelling simultaneous Poisson processes with RjMCMC could be explored. Of particular interest, if any parallels between the concurrence of specific breakpoint locations could be found, it may be of social value to determine whether or not a change in legislation had a meaningful impact on a particular crime.

In a somewhat analogous fashion, there have been debates both historical and contemporary as to whether or not severe punishment has a deterrant effect. The debate over capital punishment was explored in 1910 in such brazen musings as “Why should they not be eliminated once for all?” (MacDonald 1910). In the early 20th century one argument to retain the death penalty was that if the penalty for theft and robbery was the same as for homicide, then what motivation would a criminal have to stop short of murder when the alternative would rid them of a pesky witness to their crimes (MacDonald 1910).

There are many directions that further research in this topic could take. One way of dealing with the multiple breakpoints and also with significant changes in rate of change could be to use a non-parametric regression approach such as multivariate adaptive regression splines, under which the number of knots could be adjusted for spline smoothing in accordance with the predicted number of breakpoints. One reason that I did not employ a non-parametric approach was due to the relative lack of data in the problem, as it would become difficult to partition the dataset into training and validation subsets.

If the scope of this project were to be extended, I would aim to develop the model to handle multiple sets of data, for example, comparing whether the conviction rates for men and women were similar, and whether the changes in rates occurred at a similar time period. If this analysis were expanded to model different crimes and the associated punishments, the resulting model could possibly point to key changes in regime or global events that influence the underlying behavior of crime in England.

While some legislative changes have been roughly addressed, changes in the media have not been examined in great detail. This research cannot easily be extended to the modern interpretations of crime as the landscape of the trial has shifted - definitions of each crime have changed, the severity of punishments have been altered, and trials go for weeks or months, as opposed to the 30 minutes suggested by Beattie (2001b). In saying this, it could be of interest to investigate the effect that reporting has on crime, such as in the context of the debate over whether or not to name suspects accused of terrorism or mass murder.

Another direction that could be explored is also whether or not there is evidence to support a separate judicial system for religious crimes, particularly in light of the abuse scandals of the Catholic Church that have plagued the religion in recent times.

In a more general way, if any of these potential avenues were followed, a larger study could be undertaken to determine what effect, if any, justice reform has on different crimes. Regardless of the future prospects of this research, it is without question that the conversations surrounding the links between media, crime, punishment, and morality will continue to occur in our modern age of social media, the rise and fall of the 24-hour news cycle, and the re-emergence of “fake news” and “alternative facts.”

6 Appendix: Supplementary Figures and Tables

In this section, each category of crime or punishment is considered by displaying the historical data, the fixed k MH estimates, and the results of the RjMCMC algorithm. For the tables of estimates for s, the locations of breakpoints, the posterior estimates and 95% CIs are given in terms of the data on $[0, L]$, with an additional column denoting the approximate year that is represented.

Figures and Tables - Crimes by Offence - All Crimes - (Old Bailey Online 2018a)

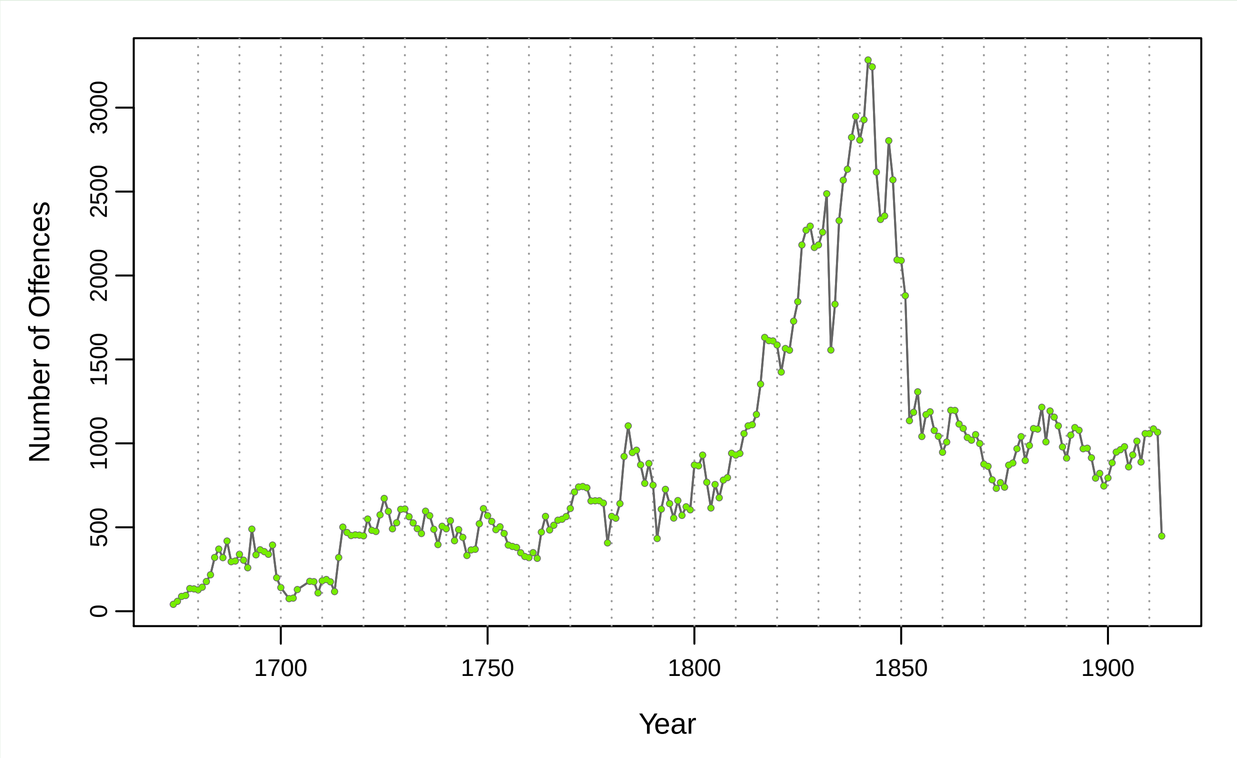


Figure 7: Number of Crimes heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

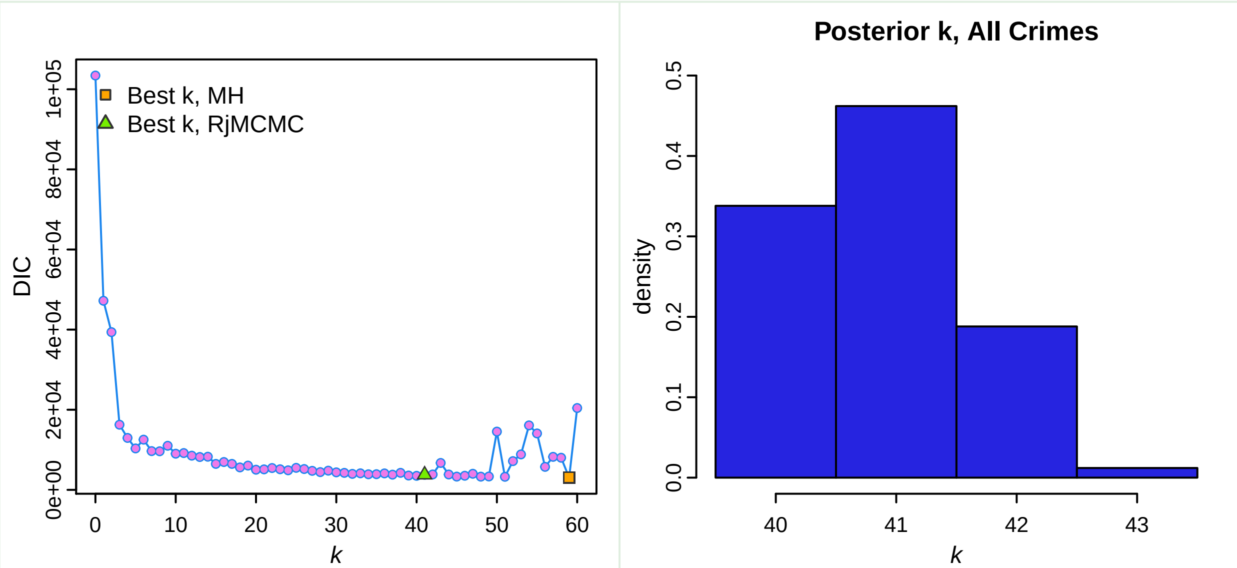


Figure 8: Model Estimates - DIC vs. Posterior k, All Crimes by Offence (Old Bailey Online 2018a).

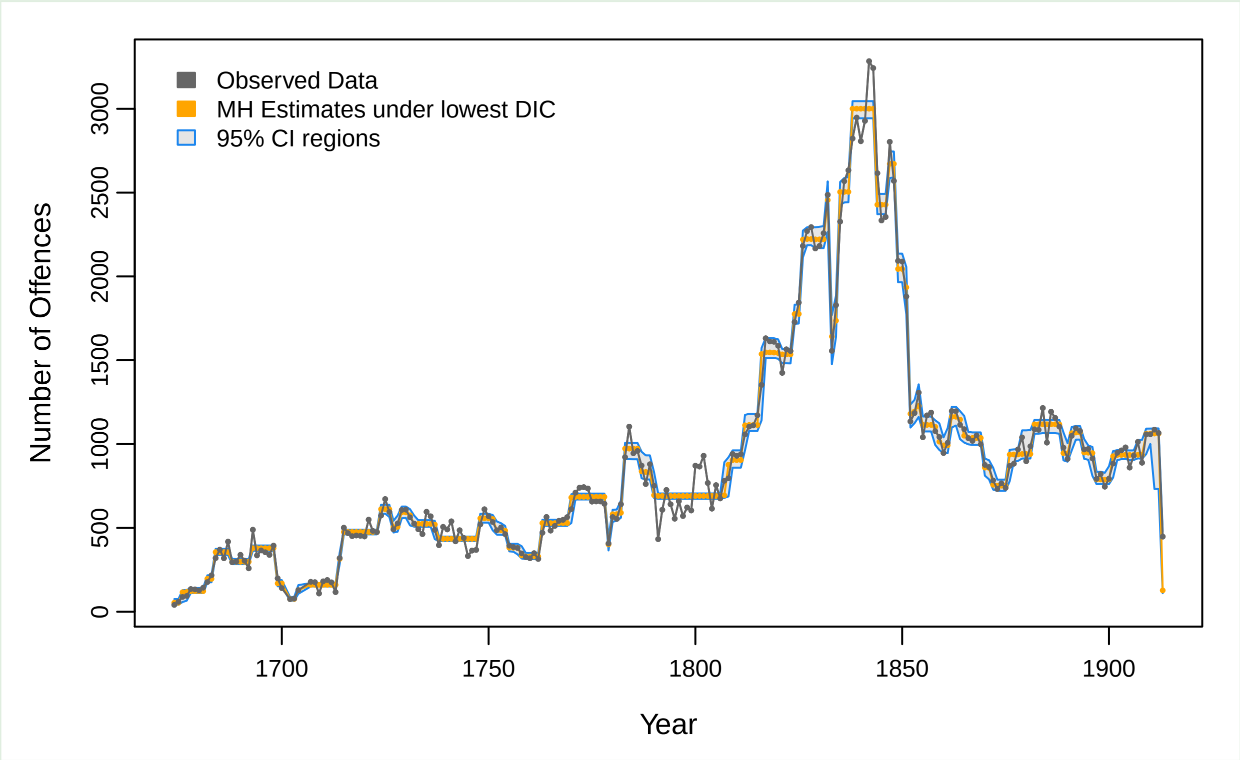


Figure 9: MH Estimates per Year, All Crimes by Offence (Old Bailey Online 2018a).

k	DIC
0	103426.3
1	47201.1
2	39369.3
3	16261.7
4	12960.7
5	10335.5
6	12526.9
7	9672.9
8	9630.1
9	11003.1
10	9026.0
11	9180.9
12	8531.7
13	8194.7
14	8273.2
15	6481.2
16	6944.7
17	6479.4
18	5600.0
19	6054.3
20	5028.0
21	5114.2
22	5446.5
23	5142.6
24	4879.7
25	5494.7
26	5196.8
27	4743.4
28	4427.0
29	4788.1
30	4370.1
31	4237.7
32	3987.2
33	4124.3
34	3888.0
35	3909.1
36	4107.4
37	3793.2
38	4244.7
39	3573.7
40	3498.7
41	3762.7
42	3842.0
43	6713.5
44	3832.3
45	3305.3
46	3520.5
47	4013.2
48	3293.4
49	3336.0
50	14538.0
51	3260.7
52	7170.7
53	8846.8
54	16095.3
55	14081.5
56	5740.8
57	8240.5
58	8015.3
59	3042.0
60	20426.1

Table 6: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	53.6	(41.2, 75.3)
1	122.3	(111.3, 141.2)
2	196.2	(175.6, 217.4)
3	355.5	(338.3, 374.9)
4	298.1	(283.7, 314)
5	379	(363.5, 395)
6	169	(151.6, 188.5)
7	76	(65.5, 87.5)
8	128.4	(107, 158.8)
9	160.4	(150.7, 170.9)
10	315.1	(284.9, 352.9)
11	475.7	(461.5, 489.3)
12	611.5	(584.8, 638.5)
13	505	(472.5, 542.6)
14	592.6	(559.5, 627.5)
15	524.1	(506.9, 545.9)
16	435.1	(421.7, 448.1)
17	557	(531.7, 584.2)
18	483.8	(456, 511.7)
19	382	(360.3, 405.2)
20	329.6	(312.9, 349.7)
21	528.6	(512, 548.7)
22	685.1	(669.6, 704.8)
23	401.2	(365.5, 445)
24	581.6	(536.9, 608.6)
25	972.9	(605.6, 1007.7)
26	833.6	(788.9, 932.7)
27	690.4	(673.8, 705.5)
28	904.5	(859.5, 962.4)
29	1113.9	(1078, 1180.2)
30	1546.8	(1513.5, 1633.8)
31	1756.8	(1481, 1827.4)
32	2202.9	(1727.6, 2268.2)
33	2250.7	(2174.5, 2547)
34	2376.7	(1476, 2557.3)
35	1736.3	(1638.8, 1884.8)
36	2504.2	(2438.6, 2566.2)
37	3000.7	(2943.3, 3045.8)
38	2428	(2371.3, 2492.9)
39	2671.6	(2590.6, 2745.1)
40	2044.7	(1965, 2136.1)
41	1773.9	(1098.4, 1934.7)
42	1227.3	(1162.1, 1356.3)
43	1112.7	(1049.9, 1152.1)
44	992.4	(945.6, 1039.9)
45	1163.1	(1110.8, 1222.8)
46	1036.4	(995.1, 1069.5)
47	858.7	(804.7, 905.3)
48	751.7	(721.4, 788.1)
49	937.9	(897, 968.2)
50	1118	(1064.9, 1144.8)
51	944.6	(894.9, 1000.8)
52	1068.9	(1026.7, 1107.8)
53	948.4	(908.7, 985.2)
54	789.1	(761.1, 835.2)
55	929.7	(800.4, 964.2)
56	1046.8	(889, 1091.6)
57	475.5	(405.8, 1088)
58	60.4	(41.2, 479.7)
59	113.3	(46.1, 130.9)

Table 7: MH Posterior Estimates for h, conditioned on k = 59

s	Posterior Estimate	95% CI	Year
1	2.7	(2, 4.8)	1677
2	8.4	(8, 9)	1683
3	10.5	(10, 11)	1685
4	14.5	(14, 15)	1689
5	19.6	(19, 20)	1694
6	25.5	(25, 26)	1700
7	27.9	(27.1, 29)	1702
8	30.5	(30, 31)	1705
9	32.7	(31.2, 33.9)	1707
10	40.4	(40, 41)	1715
11	41.6	(41.1, 42)	1716
12	50.5	(50, 51)	1725
13	53.4	(52.9, 53.9)	1728
14	55.5	(54.5, 56)	1730
15	58.3	(57, 59.6)	1733
16	64.4	(63.1, 65)	1739
17	74.5	(74, 75)	1749
18	78.5	(77.3, 80.1)	1753
19	81.5	(81, 82)	1756
20	84.8	(83.6, 86)	1759
21	89.6	(89, 90)	1764
22	96.7	(96, 97.9)	1771
23	105.4	(105, 105.9)	1780
24	106.5	(106, 107)	1781
25	109.4	(108.1, 110)	1784
26	113.4	(109.2, 114.3)	1788
27	116.7	(116, 117.9)	1791
28	134.7	(133.2, 136)	1809
29	138.5	(138.1, 139.6)	1813
30	142.7	(142.1, 143.7)	1817
31	150.3	(146, 150.9)	1825
32	152.2	(150.1, 153)	1827
33	155.5	(152.2, 158.9)	1830
34	158.9	(158.1, 160)	1833
35	159.9	(159, 161)	1834
36	161.6	(161, 162)	1836
37	164.5	(164, 165)	1839
38	170.5	(170, 170.9)	1845
39	173.4	(173.1, 174)	1848
40	175.5	(175.1, 176)	1850
41	178	(177.1, 178.9)	1852
42	179	(178.1, 180.8)	1853
43	181.6	(181.1, 184.5)	1856
44	185.6	(184.2, 186.9)	1860
45	188.5	(188, 189)	1863
46	191.5	(190.2, 192.9)	1866
47	196.5	(196, 197)	1871
48	198.7	(197.9, 199.7)	1873
49	202.4	(202, 203.3)	1877
50	208.5	(205.1, 209)	1883
51	215.6	(215, 216.4)	1890
52	217.5	(217, 218.6)	1892
53	220.6	(220, 221.4)	1895
54	223.4	(222.3, 224)	1898
55	227.4	(226.3, 228.8)	1902
56	235.1	(228, 236)	1910
57	239	(233.9, 239.6)	1913
58	239.5	(239, 239.9)	1913
59	240	(239.6, 240.7)	1913

Table 8: MH Posterior Estimates for s, conditioned on k = 59

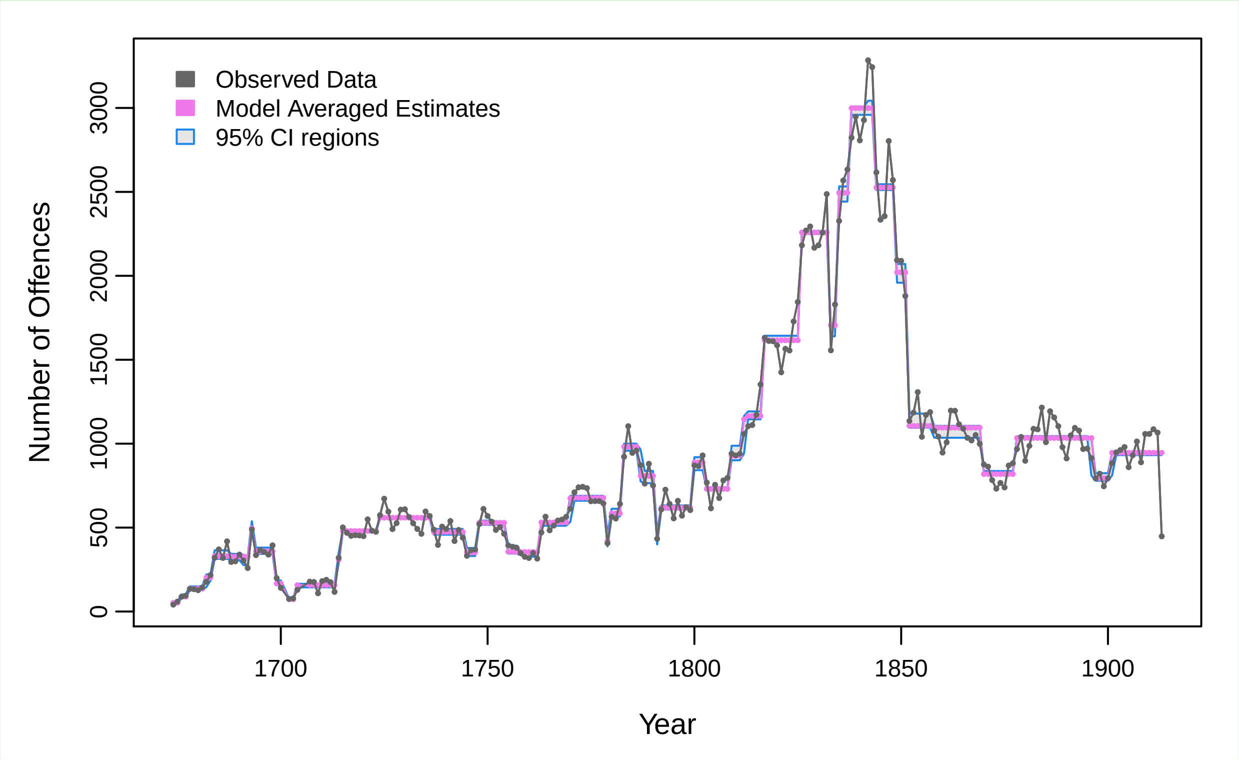


Figure 10: Model Averaged Estimates per Year, All Crimes by Offence (Old Bailey Online 2018a).

k	Proportion
40	0.338
41	0.462
42	0.188
43	0.012

Table 9: Posterior estimate for k

h	Posterior Estimate	95% CI
0	52	(43.6, 58.1)
1	86.7	(80.4, 96.5)
2	133.3	(127.8, 145.6)
3	202.4	(183.7, 221.2)
4	327.5	(315.3, 364.7)
5	488	(282.4, 529.7)
6	364.4	(343.3, 538.2)
7	168.8	(157.2, 368.8)
8	72.7	(68.1, 171.2)
9	157.1	(69.4, 165)
10	286.6	(143.9, 325.7)
11	479	(289.7, 487.7)
12	558.7	(471.8, 558.7)
13	482.5	(456.3, 558.7)
14	361.1	(330.8, 478.4)
15	528.6	(351.8, 536.4)
16	358.5	(345.8, 541.4)
17	511.1	(326.5, 533.1)
18	659.9	(525.8, 677.4)
19	420.9	(405.3, 689.7)
20	566.4	(393.8, 600.8)
21	971.6	(584.2, 994.9)
22	809	(788.8, 1024.4)
23	441.9	(416.8, 813.1)
24	608.5	(399.6, 620.2)
25	842.1	(610, 891.7)
26	730.5	(728.1, 919.7)
27	925.6	(725.5, 944.4)
28	1164.8	(920.2, 1164.8)
29	1615.8	(1144.8, 1618.9)
30	2258.4	(1614.4, 2266.5)
31	1662.2	(1639.8, 2258.4)
32	2494.6	(1669.3, 2494.6)
33	2998.9	(2464.4, 3007)
34	2525.6	(2525.6, 3043.1)
35	2020.8	(1968.3, 2545.6)
36	1179.4	(1155.7, 1999.1)
37	1073.9	(1035.2, 1105.9)
38	819	(819, 836.8)
39	1033.3	(1032.8, 1044.8)
40	794.5	(785.3, 824.2)
41	945.9	(931.8, 946.4)

Table 10: Posterior Estimates for h, conditioned on k = 41

s	Posterior Estimate	95% CI	Year
1	2.2	(2, 2.6)	1677
2	4.5	(4.1, 4.8)	1679
3	8.5	(8, 8.8)	1683
4	10.5	(10, 10.9)	1685
5	19.7	(14.3, 19.8)	1694
6	20.2	(19.1, 20.9)	1695
7	25.3	(20.2, 25.7)	1700
8	27.9	(25.1, 28.8)	1702
9	30.4	(28.3, 30.8)	1705
10	40.6	(30.5, 41)	1715
11	41.6	(40.3, 42)	1716
12	50.4	(41, 50.8)	1725
13	63.2	(50.1, 63.9)	1738
14	71.6	(63.3, 72)	1746
15	74.5	(71.1, 74.8)	1749
16	81.2	(74.5, 81.9)	1756
17	89.5	(81, 89.8)	1764
18	96.2	(89.5, 96.9)	1771
19	105.2	(96.2, 105.3)	1780
20	106.3	(105.1, 106.7)	1781
21	109.6	(106.2, 110)	1784
22	113.6	(109.7, 114.9)	1788
23	117.4	(113, 117.8)	1792
24	118.4	(117.3, 118.8)	1793
25	126.2	(118.1, 126.7)	1801
26	129.1	(126.1, 129.8)	1804
27	135.3	(129.2, 136)	1810
28	138.3	(135.1, 139)	1813
29	143.4	(138.2, 143.9)	1818
30	152.4	(143.2, 152.4)	1827
31	159.2	(152.1, 160)	1834
32	161.4	(159, 161.9)	1836
33	164	(161.1, 164.8)	1838
34	170	(164, 170.2)	1844
35	175.5	(170, 175.7)	1850
36	178.1	(175.2, 178.6)	1853
37	184.5	(178.2, 184.8)	1859
38	196.6	(196, 197)	1871
39	204.4	(204, 204.9)	1879
40	223.1	(222.2, 223.8)	1898
41	227.7	(227.1, 228.8)	1902

Table 11: Posterior Estimates for s, conditioned on k = 41

Figures and Tables - Crimes by Offence - Killing - (Old Bailey Online 2018a)

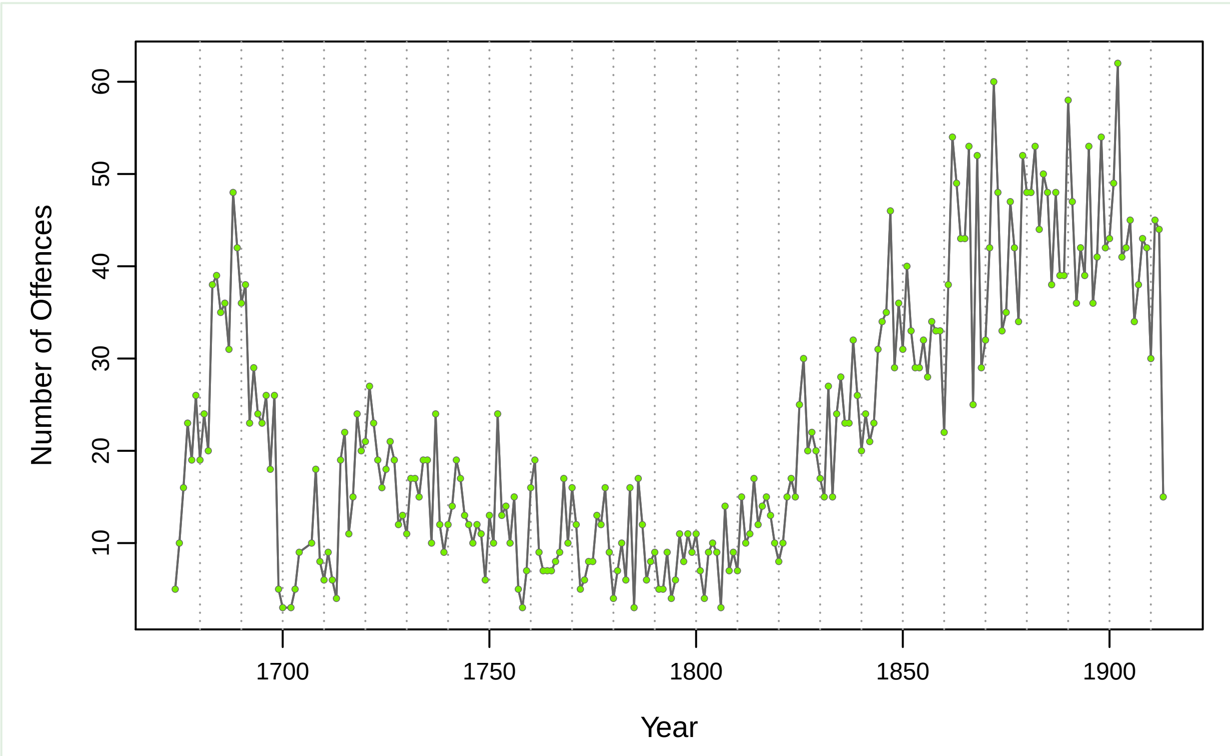


Figure 11: Number of Killing offences heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

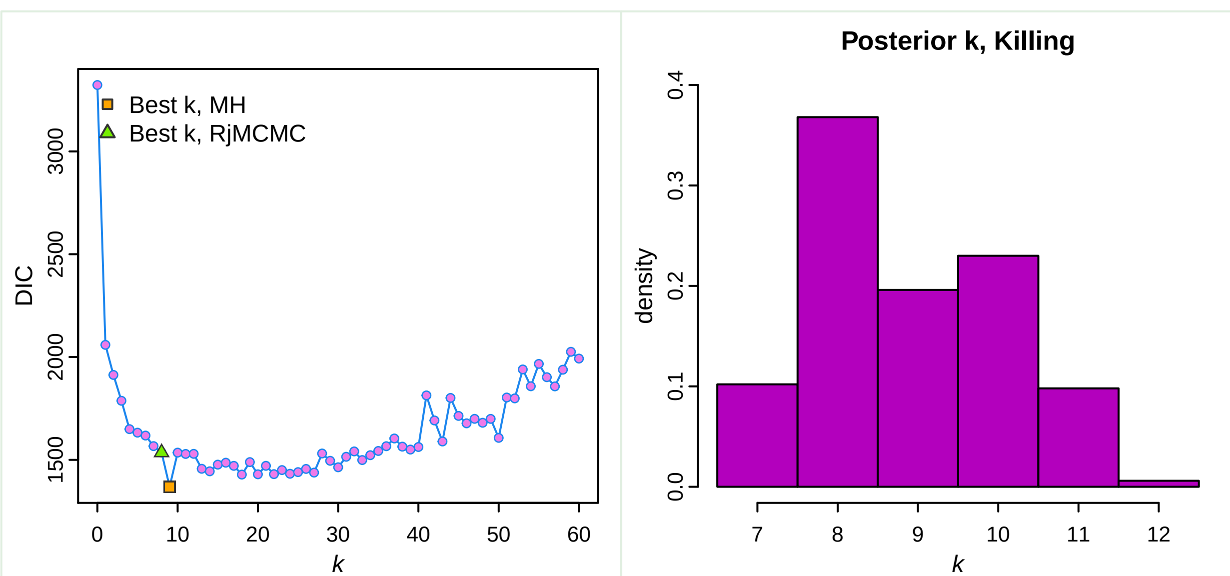


Figure 12: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Killing (Old Bailey Online 2018a).

Figures and Tables - Crimes by Offence - Killing - MH - (Old Bailey Online 2018a)

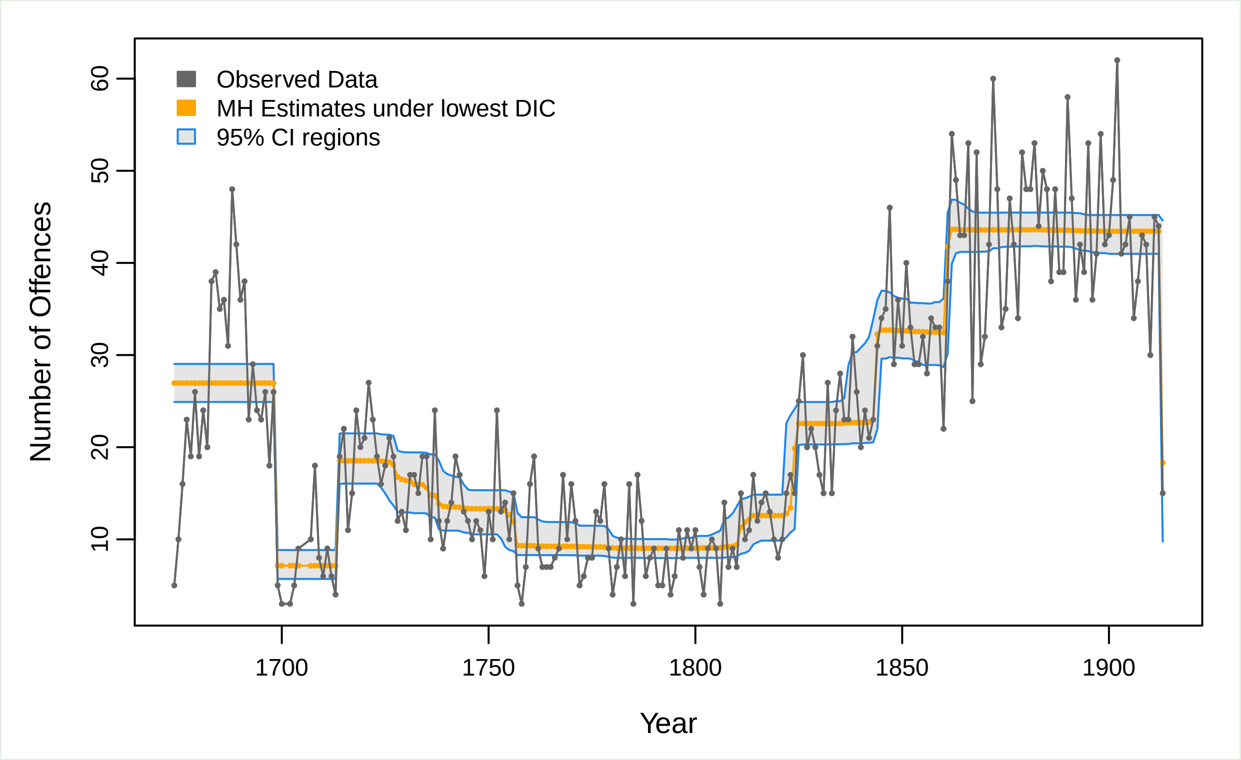


Figure 13: MH Estimates per Year, Crimes by Offence, Killing (Old Bailey Online 2018a).

k	DIC
0	3322.8
1	2059.2
2	1913.0
3	1787.3
4	1649.4
5	1632.2
6	1618.1
7	1566.7
8	1535.4
9	1369.0
10	1535.1
11	1528.8
12	1529.3
13	1456.6
14	1444.3
15	1476.7
16	1486.2
17	1470.9
18	1428.3
19	1489.1
20	1429.7
21	1471.0
22	1430.5
23	1450.3
24	1432.3
25	1440.3
26	1455.8
27	1438.0
28	1530.9
29	1495.4
30	1463.8
31	1514.9
32	1541.0
33	1499.0
34	1522.6
35	1543.2
36	1566.3
37	1604.0
38	1564.2
39	1549.9
40	1562.5
41	1813.4
42	1691.6
43	1589.3
44	1801.1
45	1713.7
46	1677.6
47	1699.5
48	1680.4
49	1699.3
50	1607.0
51	1803.2
52	1798.9
53	1939.6
54	1857.8
55	1966.6
56	1901.9
57	1857.3
58	1938.4
59	2025.4
60	1992.4

Table 12: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	27	(24.9, 29)
1	7.1	(5.7, 8.8)
2	18.5	(16.1, 21.5)
3	13.3	(10.5, 15.3)
4	9	(8, 10)
5	12.6	(9.9, 14.8)
6	22.6	(20.3, 24.9)
7	32.7	(29.7, 37)
8	43.6	(28.8, 47.3)
9	18.3	(9.8, 44.6)

Table 13: MH Posterior Estimates for h, conditioned on k = 9

s	Posterior Estimate	95% CI	Year
1	25.5	(25, 26)	1700
2	40.5	(40, 41)	1715
3	61.7	(51.4, 70.8)	1736
4	83.4	(80.3, 107.3)	1758
5	137.5	(123.9, 141)	1812
6	150.9	(148.4, 151.9)	1825
7	170.5	(163.9, 171.9)	1845
8	187.8	(175.5, 188.9)	1862
9	239.1	(187.4, 239.9)	1913

Table 14: MH Posterior Estimates for s, conditioned on k = 9

Figures and Tables - Crimes by Offence - Killing - RjMCMC - (Old Bailey Online 2018a)

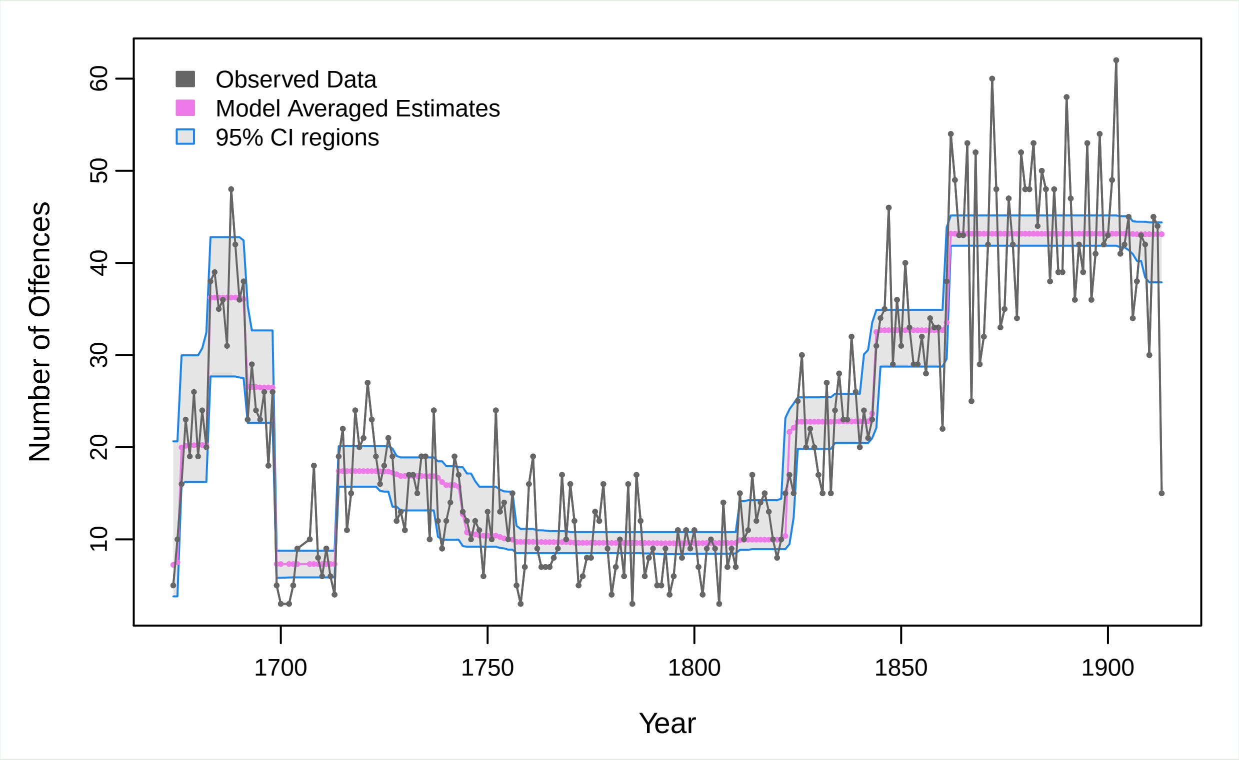


Figure 14: Model Averaged Estimates per Year, Crimes by Offence, Killing (Old Bailey Online 2018a).

k	Proportion
7	0.102
8	0.368
9	0.196
10	0.230
11	0.098
12	0.006

Table 15: Posterior estimate for k

h	Posterior Estimate	95% CI
0	16.9	(3.7, 20.6)
1	35.1	(18.6, 42.4)
2	26.7	(8, 32.9)
3	7	(5.9, 18.1)
4	16.7	(9.8, 18)
5	9.8	(8.9, 10.8)
6	22.6	(19.6, 24.3)
7	32.5	(29, 34.9)
8	43.6	(41.7, 44.2)

Table 16: Posterior Estimates for h, conditioned on k = 8

s	Posterior Estimate	95% CI	Year
1	9	(2, 9.9)	1683
2	18.4	(7.1, 25.7)	1693
3	25.6	(25.1, 40.8)	1700
4	40.6	(40, 70.2)	1715
5	71.2	(65.4, 134)	1746
6	149.3	(148, 151.6)	1824
7	170.4	(167, 171.6)	1845
8	188.1	(187.4, 189)	1863

Table 17: Posterior Estimates for s, conditioned on k = 8

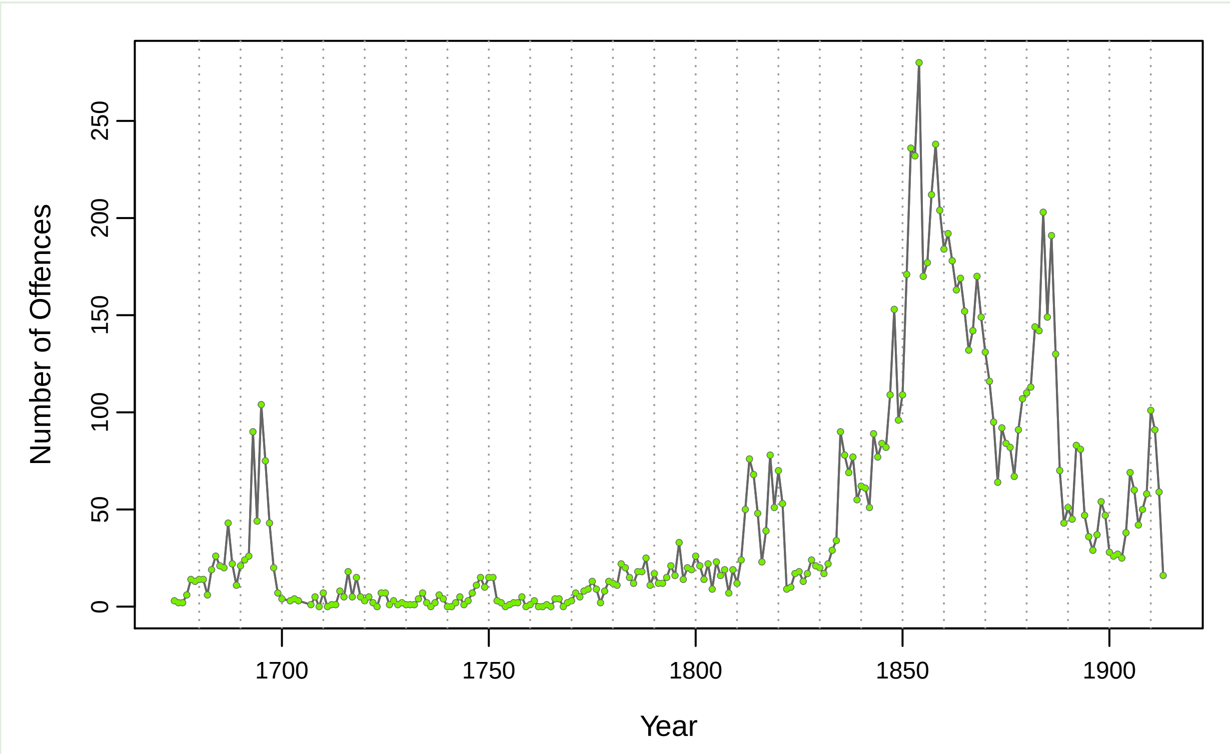


Figure 15: Number of Royal Offences heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

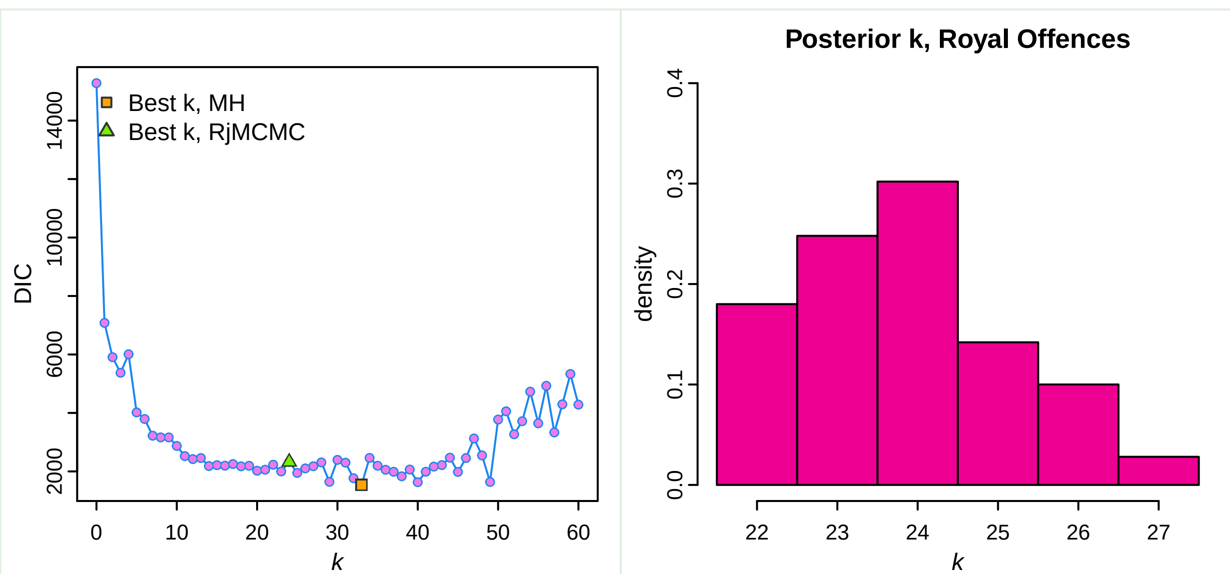


Figure 16: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Royal Offences (Old Bailey Online 2018a).

Figures and Tables - Crimes by Offence - Royal Offences - MH - (Old Bailey Online 2018a)

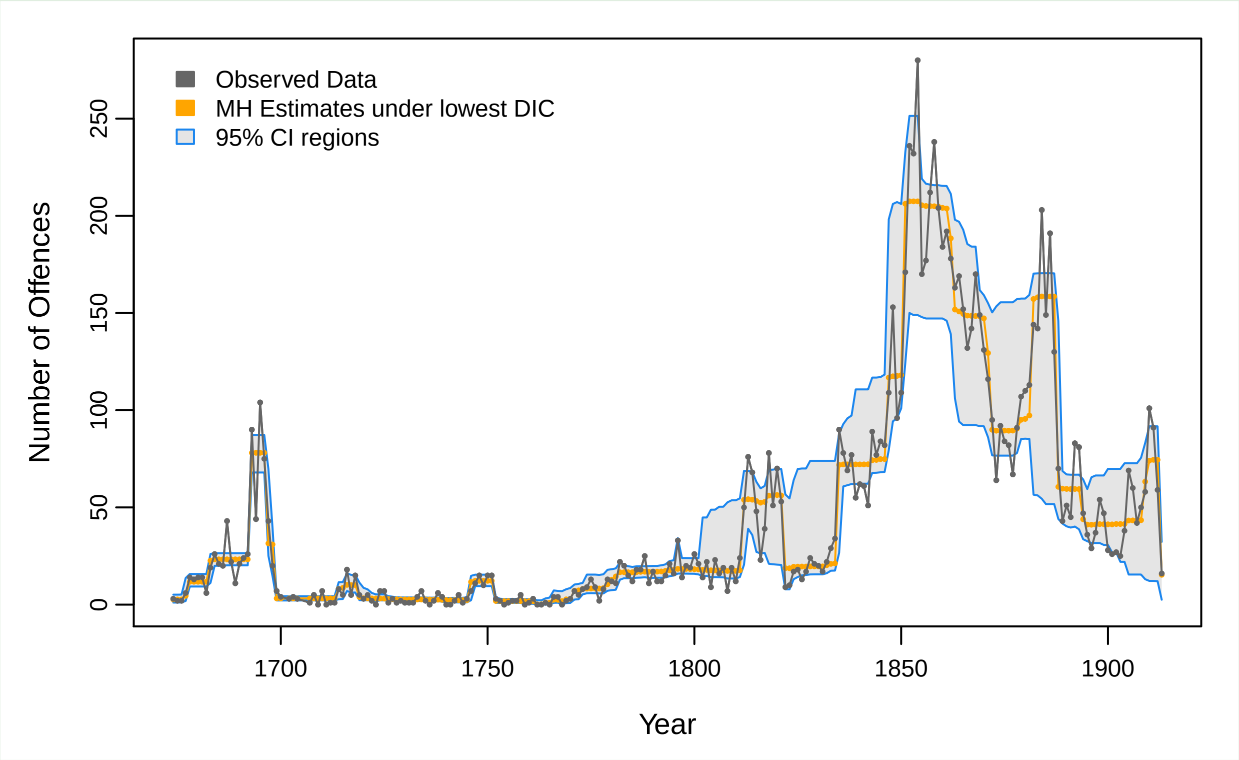


Figure 17: MH Estimates per Year, Crimes by Offence, Royal Offences (Old Bailey Online 2018a).

k	DIC
0	15280.8
1	7079.8
2	5906.9
3	5373.3
4	6007.5
5	4018.1
6	3792.8
7	3220.9
8	3163.3
9	3162.9
10	2871.7
11	2523.2
12	2421.4
13	2457.2
14	2183.5
15	2214.1
16	2195.7
17	2251.5
18	2173.8
19	2192.7
20	2023.6
21	2057.8
22	2228.9
23	1997.9
24	2323.3
25	1948.5
26	2102.6
27	2176.4
28	2308.3
29	1642.9
30	2394.5
31	2299.8
32	1767.5
33	1537.2
34	2461.0
35	2192.7
36	2055.7
37	1988.1
38	1832.6
39	2063.8
40	1630.6
41	1988.7
42	2163.5
43	2216.2
44	2468.9
45	1981.9
46	2457.3
47	3126.2
48	2546.5
49	1638.3
50	3774.0
51	4055.2
52	3269.7
53	3715.9
54	4728.4
55	3644.5
56	4927.9
57	3332.4
58	4295.0
59	5332.5
60	4282.3

Table 18: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	2.7	(1.1, 5.1)
1	11.8	(9.3, 15.8)
2	23.3	(20.2, 26.5)
3	78.1	(68, 87.3)
4	31	(14.6, 39.5)
5	3.2	(2.1, 4.3)
6	10.2	(7, 16.1)
7	3.1	(1.5, 5.3)
8	2.3	(1.1, 3.5)
9	12.3	(9.5, 15.4)
10	1.9	(1, 3.4)
11	0	(0, 2.5)
12	1.5	(0, 6.5)
13	2.2	(0, 8.5)
14	7.2	(1.3, 16.4)
15	13.1	(6.4, 21.5)
16	17.2	(14.1, 20.4)
17	21.6	(14.7, 64.8)
18	19.7	(8.8, 68.4)
19	26.2	(10.5, 68.2)
20	55.3	(9.7, 73)
21	26.8	(17.5, 90.3)
22	77.4	(68.7, 138.9)
23	121.9	(104.2, 251.4)
24	202.4	(146.5, 214.9)
25	148.2	(91.7, 160.5)
26	90.4	(76.7, 150.3)
27	119.6	(59.7, 168.6)
28	137.4	(45.9, 169.8)
29	53.1	(32.6, 66.8)
30	43.3	(15.5, 72.7)
31	74.5	(3.3, 91.7)
32	12.8	(1.7, 23.6)
33	3.5	(1.2, 23.6)

Table 19: MH Posterior Estimates for h, conditioned on k = 33

s	Posterior Estimate	95% CI	Year
1	4.2	(3, 5)	1679
2	9.6	(9.1, 11)	1684
3	19.4	(19, 20)	1694
4	23.4	(23, 24.8)	1698
5	25.5	(25, 26)	1700
6	40.8	(40, 42.9)	1715
7	45.9	(45, 48.8)	1720
8	61.8	(50.7, 68.5)	1736
9	72.5	(72, 73.7)	1747
10	78.5	(78, 79)	1753
11	88.2	(79.6, 89.3)	1763
12	90.5	(86.1, 92.9)	1765
13	94.2	(91.9, 97.8)	1769
14	97.2	(95, 107)	1772
15	105	(97.1, 111.5)	1779
16	108.9	(105.3, 122.4)	1783
17	128.7	(118.9, 139)	1803
18	138.3	(123.6, 148.6)	1813
19	142.7	(137, 152)	1817
20	148.1	(138.4, 161.7)	1823
21	150.3	(146.7, 169.9)	1825
22	162.6	(161, 174)	1837
23	173.8	(173, 178.6)	1848
24	177.7	(176.9, 188.4)	1852
25	189.5	(188.1, 195.3)	1864
26	198	(194.3, 199.6)	1872
27	206.2	(198.4, 209.8)	1881
28	209.8	(208.1, 215.4)	1884
29	215.3	(214, 221.9)	1890
30	222.8	(220, 231.9)	1897
31	236.7	(235, 239.8)	1911
32	239.5	(239, 240.5)	1913
33	240.5	(239.7, 240.9)	1913

Table 20: MH Posterior Estimates for s, conditioned on k = 33

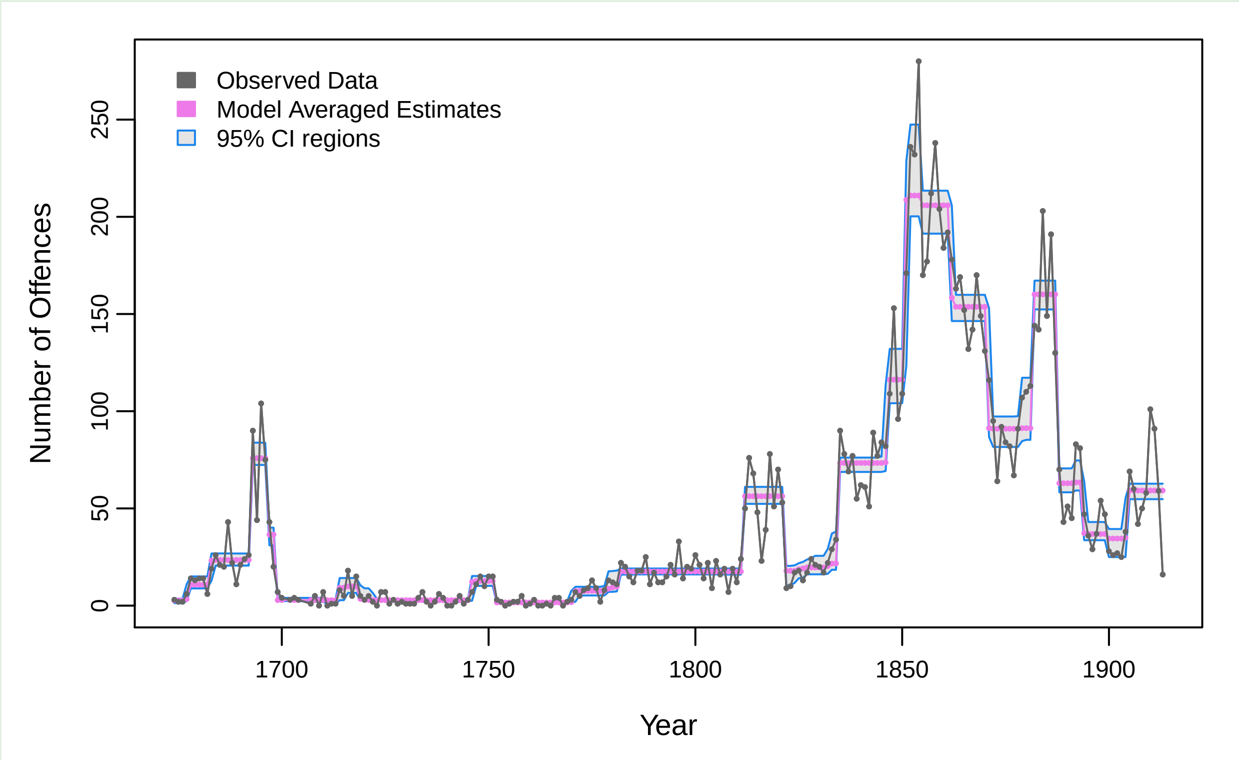


Figure 18: Model Averaged Estimates per Year, Crimes by Offence, Royal Offences (Old Bailey Online 2018a).

k	Proportion
22	0.180
23	0.248
24	0.302
25	0.142
26	0.100
27	0.028

Table 21: Posterior estimate for k

h	Posterior Estimate	95% CI
0	2.7	(1.3, 3.5)
1	11	(8.9, 15)
2	23.9	(21.2, 26.8)
3	77.2	(72.6, 82.7)
4	35.5	(28.9, 40.7)
5	2.9	(2.3, 4.6)
6	9	(0.9, 14.2)
7	2.9	(2.5, 11)
8	12	(1.9, 14.6)
9	1.8	(1.3, 14.5)
10	6.1	(1.2, 9.5)
11	17	(6.4, 18.6)
12	55.7	(15, 59.7)
13	19.5	(10.4, 59.9)
14	21.8	(14.8, 73.4)
15	73.1	(21.1, 123.4)
16	115	(68.8, 248.3)
17	200.2	(104.1, 233.9)
18	157.9	(146.8, 213.4)
19	94.6	(82, 159.2)
20	126.6	(87, 166.8)
21	153.6	(57.1, 166)
22	62.2	(38.6, 74.7)
23	34.5	(25, 39.4)
24	59.2	(55.1, 62.7)

Table 22: Posterior Estimates for h, conditioned on k = 24

s	Posterior Estimate	95% CI	Year
1	4.1	(3.2, 4.8)	1679
2	9.8	(9.4, 10.7)	1684
3	19.4	(19.1, 20)	1694
4	23.2	(23, 23.8)	1698
5	25.4	(25.2, 26)	1700
6	40.4	(30.3, 43)	1715
7	45.5	(40, 48.3)	1720
8	72.3	(45.1, 73.8)	1747
9	78.4	(72.1, 78.8)	1753
10	97.1	(78, 99)	1772
11	105	(96.6, 108.6)	1779
12	138.2	(105.5, 138.8)	1813
13	148.4	(138.1, 148.9)	1823
14	151.4	(148, 161.7)	1826
15	161.7	(151.4, 173.8)	1836
16	173.2	(161.1, 178.8)	1848
17	177.3	(172.2, 181.6)	1852
18	188.5	(177.1, 189.3)	1863
19	197.4	(188.5, 199)	1872
20	206.6	(197.2, 208.8)	1881
21	209	(208.1, 214.9)	1883
22	214.9	(214, 221.4)	1889
23	221.4	(220.2, 226.9)	1896
24	231.6	(230.6, 231.9)	1906

Table 23: Posterior Estimates for s, conditioned on k = 24

Figures and Tables - Crimes by Offence - Sexual Offences - (Old Bailey Online 2018a)

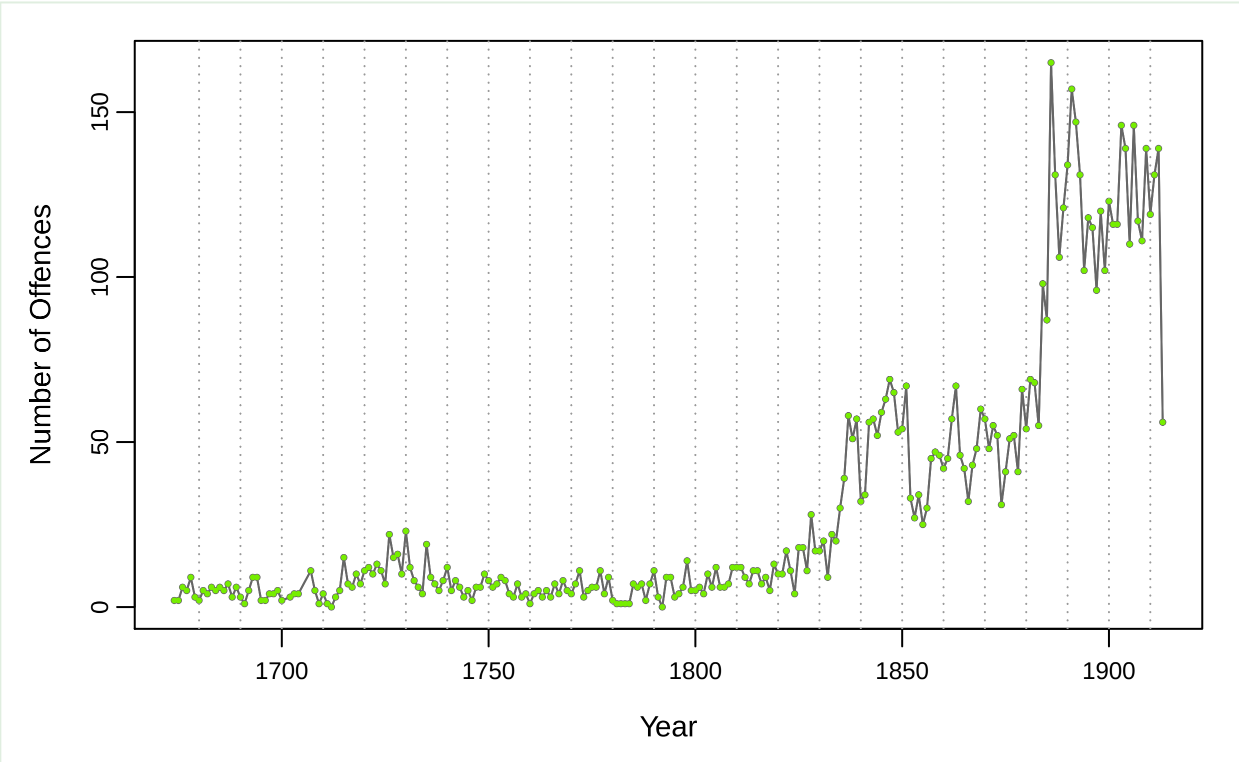


Figure 19: Number of Sexual Offences heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

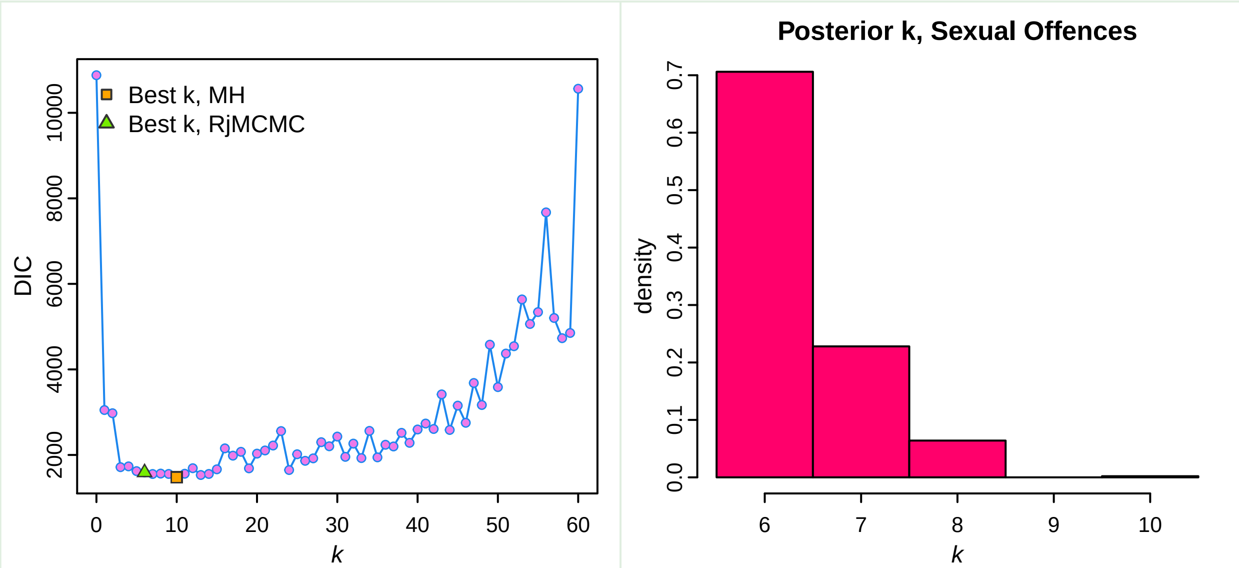


Figure 20: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Sexual Offences (Old Bailey Online 2018a).

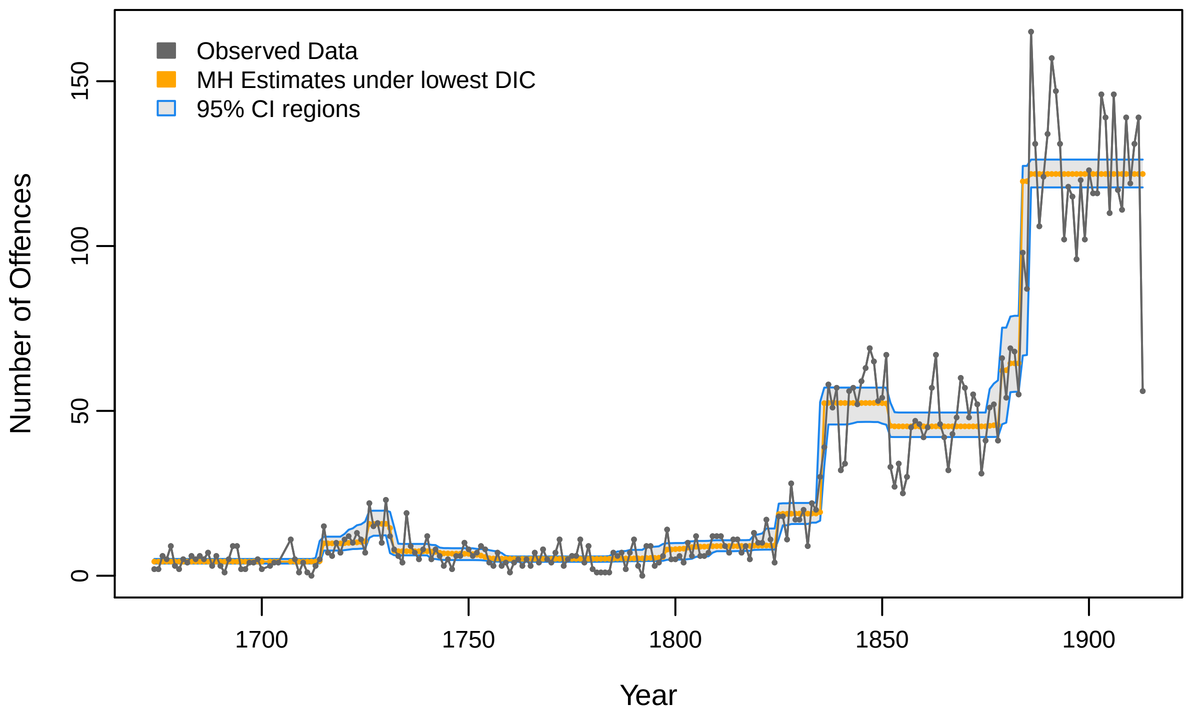


Figure 21: MH Estimates per Year, Crimes by Offence, Sexual Offences (Old Bailey Online 2018a).

k	DIC
0	10879.0
1	3050.3
2	2975.6
3	1714.2
4	1732.9
5	1622.0
6	1589.6
7	1554.5
8	1564.0
9	1555.5
10	1475.9
11	1560.7
12	1690.1
13	1531.1
14	1555.5
15	1663.8
16	2154.1
17	1983.2
18	2073.1
19	1686.7
20	2030.7
21	2104.9
22	2220.0
23	2557.9
24	1647.3
25	2016.3
26	1863.0
27	1920.3
28	2298.3
29	2203.0
30	2429.6
31	1956.0
32	2264.8
33	1927.1
34	2562.3
35	1943.7
36	2238.3
37	2198.7
38	2516.8
39	2283.2
40	2595.2
41	2734.2
42	2607.0
43	3416.4
44	2584.2
45	3153.2
46	2752.0
47	3682.6
48	3167.8
49	4578.5
50	3586.4
51	4370.6
52	4542.7
53	5636.5
54	5062.3
55	5340.3
56	7671.9
57	5202.7
58	4730.5
59	4852.4
60	10562.9

Table 24: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	4.3	(3.7, 5.1)
1	9.8	(7.6, 11.9)
2	15.7	(12.3, 19.8)
3	7.4	(6.2, 9.6)
4	5.1	(4.2, 5.9)
5	8.7	(5.5, 10.5)
6	18.2	(8.7, 21.6)
7	52.4	(17.4, 57.1)
8	45.3	(42.1, 49.5)
9	64.4	(55.8, 78.8)
10	121.9	(117.8, 126.2)

Table 25: MH Posterior Estimates for h, conditioned on k = 10

s	Posterior Estimate	95% CI	Year
1	41.3	(40, 41.9)	1716
2	52.3	(46.8, 53.4)	1727
3	58.3	(57, 59.7)	1733
4	80.4	(67.3, 85.7)	1755
5	124	(111, 131.9)	1798
6	151.3	(129.2, 152)	1826
7	162.3	(151.2, 163)	1837
8	178.2	(161.5, 179.5)	1853
9	205.5	(202.7, 207.7)	1880
10	210.9	(210, 212.9)	1885

Table 26: MH Posterior Estimates for s, conditioned on k = 10

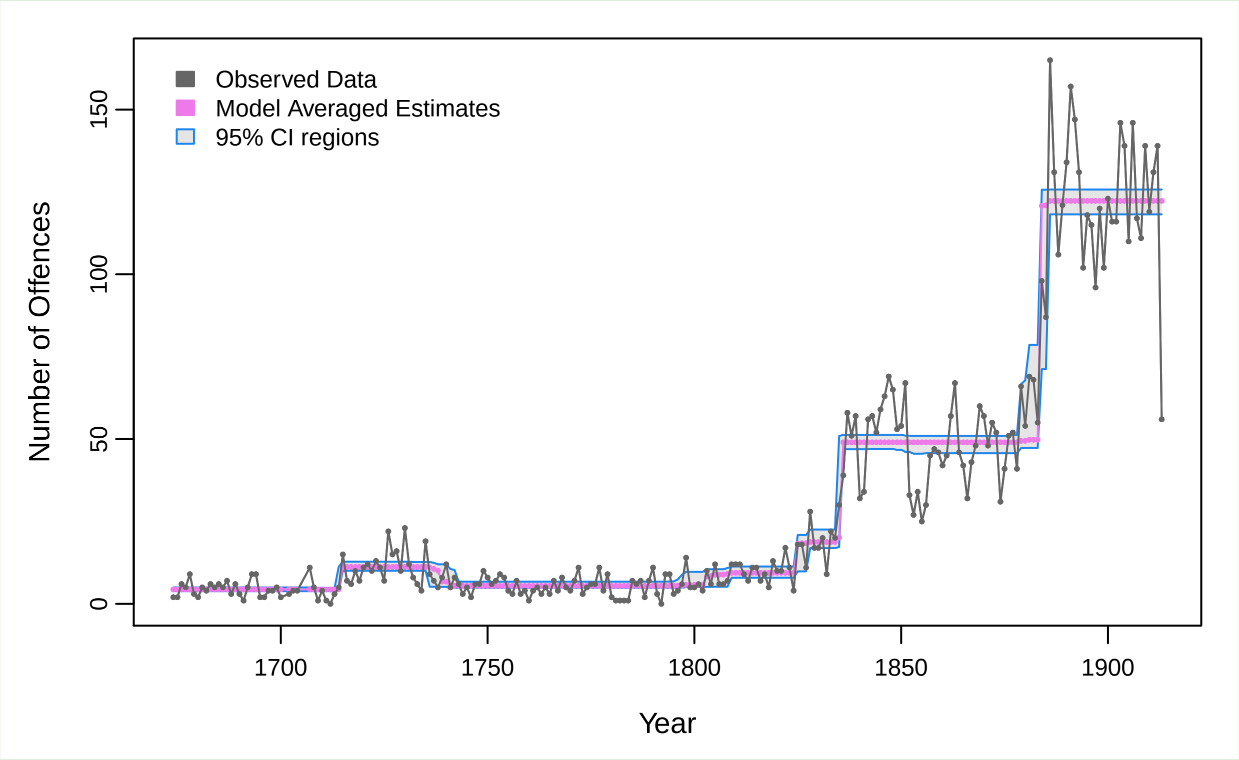


Figure 22: Model Averaged Estimates per Year, Crimes by Offence, Sexual Offences (Old Bailey Online 2018a).

k	Proportion
6	0.706
7	0.228
8	0.064
10	0.002

Table 27: Posterior estimate for k

h	Posterior Estimate	95% CI
0	4.3	(3.8, 4.8)
1	11.1	(10.1, 12.8)
2	5.5	(4.9, 6.8)
3	9.4	(7.9, 11)
4	18.8	(16.8, 22.6)
5	49.3	(47.3, 51)
6	121.3	(118.2, 127.1)

Table 28: Posterior Estimates for h, conditioned on k = 6

s	Posterior Estimate	95% CI	Year
1	41.4	(40.2, 42)	1716
2	67.3	(62.2, 69.6)	1742
3	130	(123.2, 135.9)	1804
4	151.5	(151.1, 154.5)	1826
5	162.4	(161.1, 163)	1837
6	210.5	(210.1, 211)	1885

Table 29: Posterior Estimates for s, conditioned on k = 6

Figures and Tables - Crimes by Offence - Theft - (Old Bailey Online 2018a)

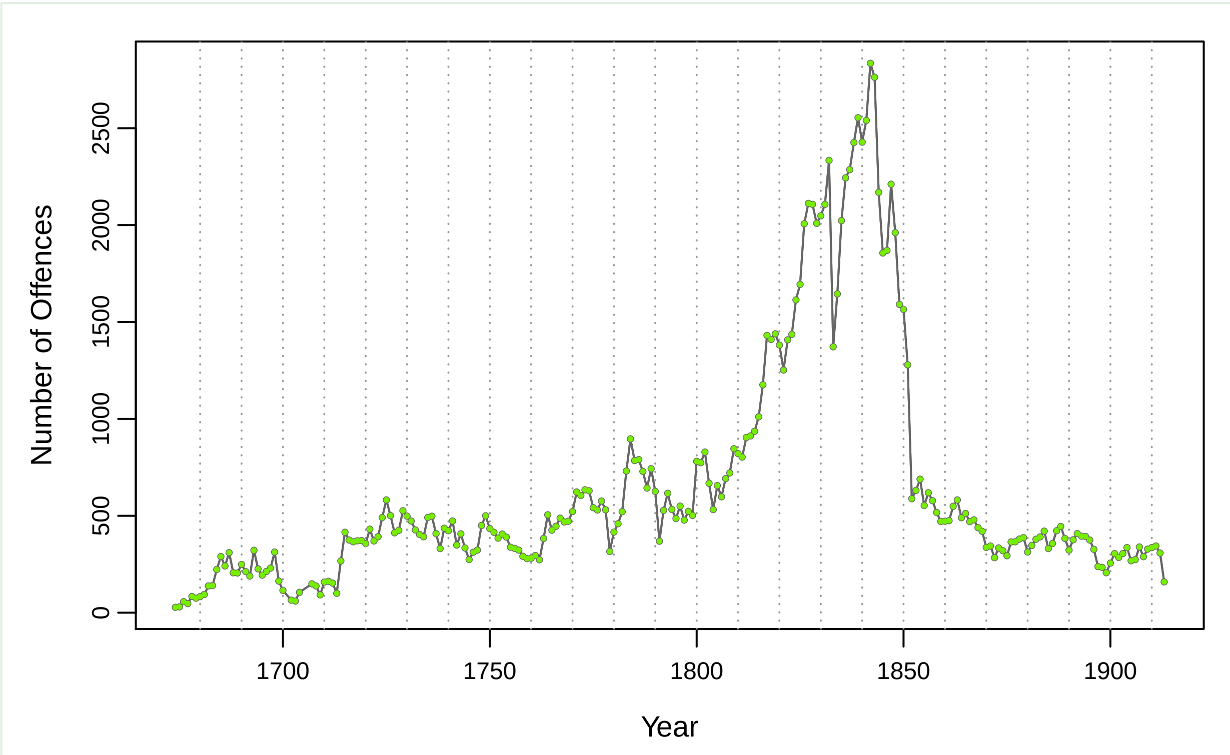


Figure 23: Number of Thefts heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

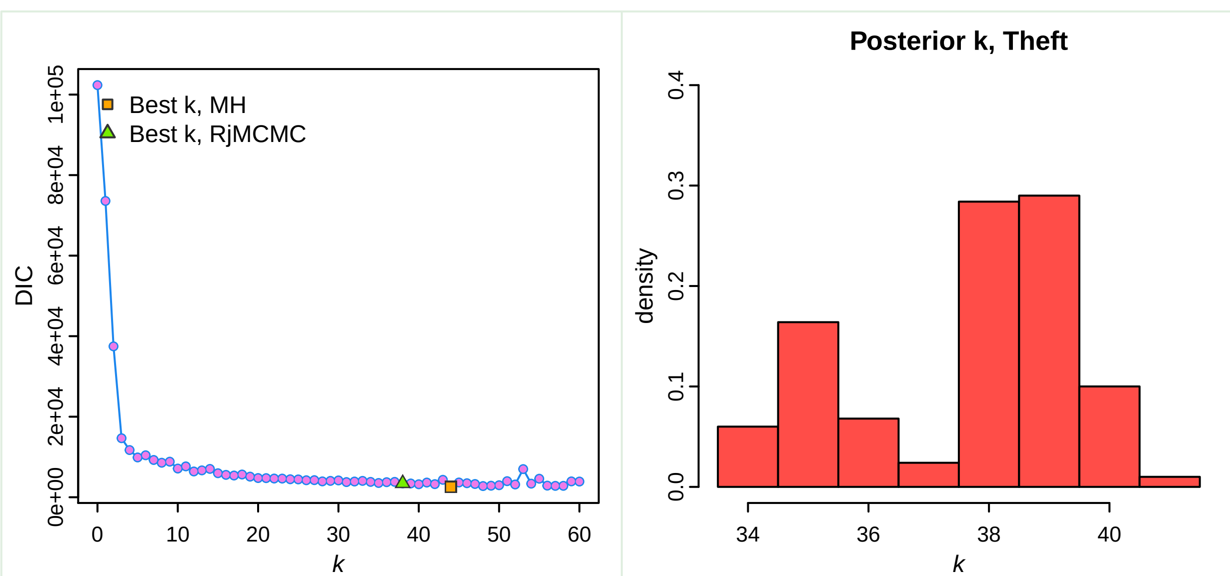


Figure 24: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Theft (Old Bailey Online 2018a).

Figures and Tables - Crimes by Offence - Theft - MH - (Old Bailey Online 2018a)

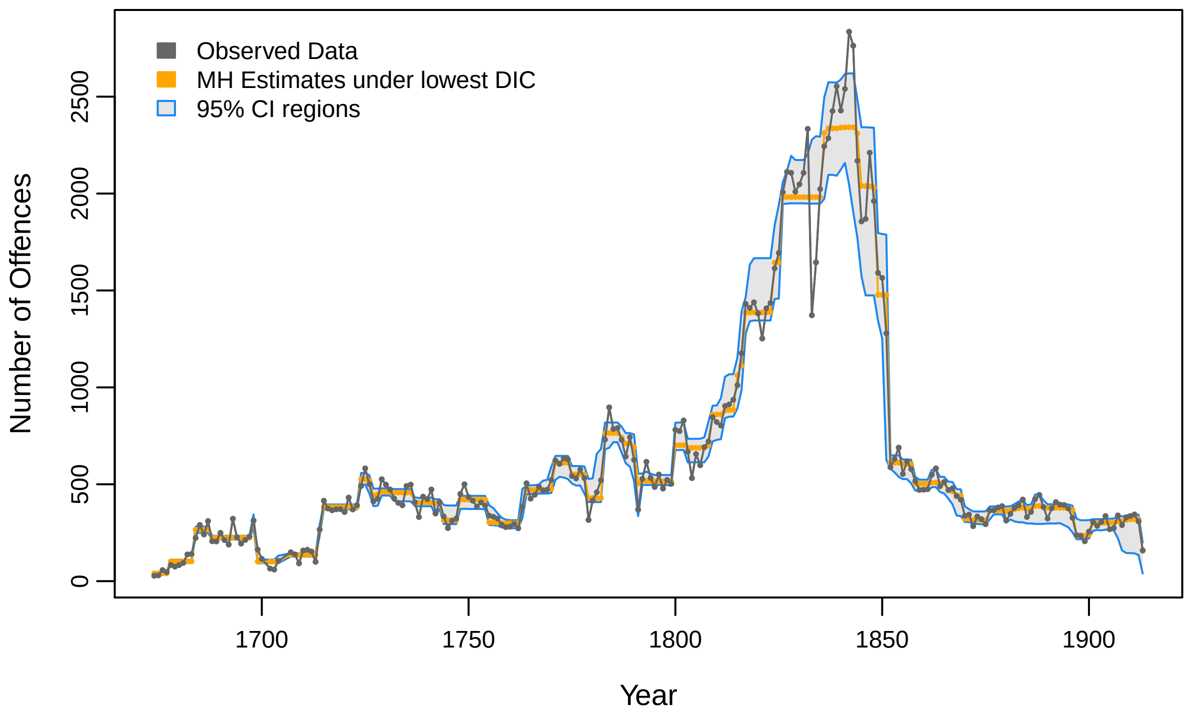


Figure 25: MH Estimates per Year, Crimes by Offence, Theft (Old Bailey Online 2018a).

k	DIC
0	102351.4
1	73582.3
2	37471.1
3	14655.0
4	11717.8
5	9883.4
6	10419.8
7	9256.1
8	8567.0
9	8831.4
10	7122.5
11	7661.2
12	6397.6
13	6670.8
14	7036.7
15	5961.5
16	5551.8
17	5389.7
18	5646.3
19	5115.5
20	4765.6
21	4746.3
22	4646.8
23	4630.6
24	4466.2
25	4426.2
26	4217.6
27	4244.0
28	3938.4
29	4059.3
30	4172.7
31	3761.0
32	3901.9
33	4059.4
34	3811.6
35	3538.5
36	3737.3
37	3814.7
38	3438.3
39	3420.3
40	3221.7
41	3645.6
42	3255.9
43	4325.3
44	2565.4
45	3663.6
46	3481.3
47	3288.8
48	2775.8
49	2859.8
50	2982.2
51	3991.4
52	3168.4
53	6986.0
54	3370.8
55	4594.7
56	2885.1
57	2825.2
58	2829.6
59	3934.8
60	3898.8

Table 30: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	40.1	(34.7, 47)
1	102.2	(95, 110.5)
2	265.5	(249.1, 281.2)
3	224.5	(215.1, 234.1)
4	310.1	(276.8, 344.1)
5	101.4	(92.9, 111.3)
6	135.8	(126.9, 144.5)
7	262.5	(235.3, 292.7)
8	383.1	(370.3, 395.7)
9	524.8	(498.8, 558.2)
10	442.6	(387.5, 477.6)
11	445.4	(390.6, 475)
12	391.9	(296.5, 426.2)
13	385	(299.1, 437.5)
14	310.8	(287.8, 437.3)
15	327.7	(273.7, 413.5)
16	466.2	(355, 492)
17	610.9	(457.5, 645)
18	551.8	(493.4, 594)
19	428.9	(407.8, 645.9)
20	764	(717.1, 818.6)
21	562.5	(335.9, 706.4)
22	515.4	(493.6, 547.7)
23	701.4	(676.7, 818.4)
24	861.2	(613.1, 944)
25	1073.4	(780.4, 1328.2)
26	1383.1	(915.7, 1423.6)
27	1628.2	(1345.1, 1735.6)
28	1968.1	(1608.7, 2119.5)
29	2311	(1964.1, 2489.6)
30	1720.2	(1422.7, 2620.1)
31	622	(587.4, 2034)
32	525.3	(485.7, 1504.1)
33	496.3	(419, 627.4)
34	433	(309.6, 568.4)
35	364.8	(293.3, 575.8)
36	379.5	(302.2, 473.4)
37	331.7	(296.4, 387.9)
38	377.2	(326.1, 447.8)
39	378.5	(297.8, 396.8)
40	240.5	(216.9, 353.7)
41	300	(216.7, 327.7)
42	312.5	(131.8, 342)
43	153.3	(37, 335.7)
44	45.5	(35.2, 179.9)

Table 31: MH Posterior Estimates for h, conditioned on k = 44

s	Posterior Estimate	95% CI	Year
1	4.6	(4, 5)	1679
2	10.4	(10, 11)	1685
3	14.6	(14.1, 15)	1689
4	24.4	(24, 25)	1699
5	25.6	(25.1, 26)	1700
6	32.3	(30.5, 33.9)	1707
7	40.4	(40, 40.9)	1715
8	41.7	(41.1, 42)	1716
9	50.4	(50, 51)	1725
10	53.4	(52.1, 54)	1728
11	56	(54.2, 64)	1730
12	64	(63.1, 71)	1738
13	70.7	(68.1, 75)	1745
14	81.2	(74.1, 83.3)	1756
15	87.7	(81.2, 89.8)	1762
16	90.1	(89, 91)	1765
17	97.2	(90.2, 98)	1772
18	101.2	(96.1, 101.9)	1776
19	105.6	(105, 107.1)	1780
20	109.5	(109, 110)	1784
21	115.1	(113.1, 117.9)	1790
22	118	(117, 124.4)	1792
23	126.5	(126, 127)	1801
24	135.2	(129.2, 138.2)	1810
25	141.2	(134, 142.7)	1816
26	143.3	(138.1, 144.4)	1818
27	150.2	(142.1, 150.9)	1825
28	152.4	(150, 153)	1827
29	162.2	(152, 163)	1837
30	171.8	(163.1, 175.9)	1846
31	178.3	(170.1, 179)	1853
32	181.9	(175.1, 184.9)	1856
33	185.2	(178.2, 194.4)	1860
34	194.3	(181.6, 196.9)	1869
35	196.4	(184.7, 202.9)	1871
36	199.1	(190.4, 210.2)	1874
37	202.5	(196, 211.9)	1877
38	207.1	(202, 214)	1882
39	213.7	(204, 216.7)	1888
40	223.2	(220.7, 224)	1898
41	227.2	(223, 233.5)	1902
42	234.7	(226.6, 239.7)	1909
43	239.4	(235, 240.5)	1913
44	240.5	(239.1, 240.9)	1913

Table 32: MH Posterior Estimates for s, conditioned on k = 44

Figures and Tables - Crimes by Offence - Theft - RjMCMC - (Old Bailey Online 2018a)

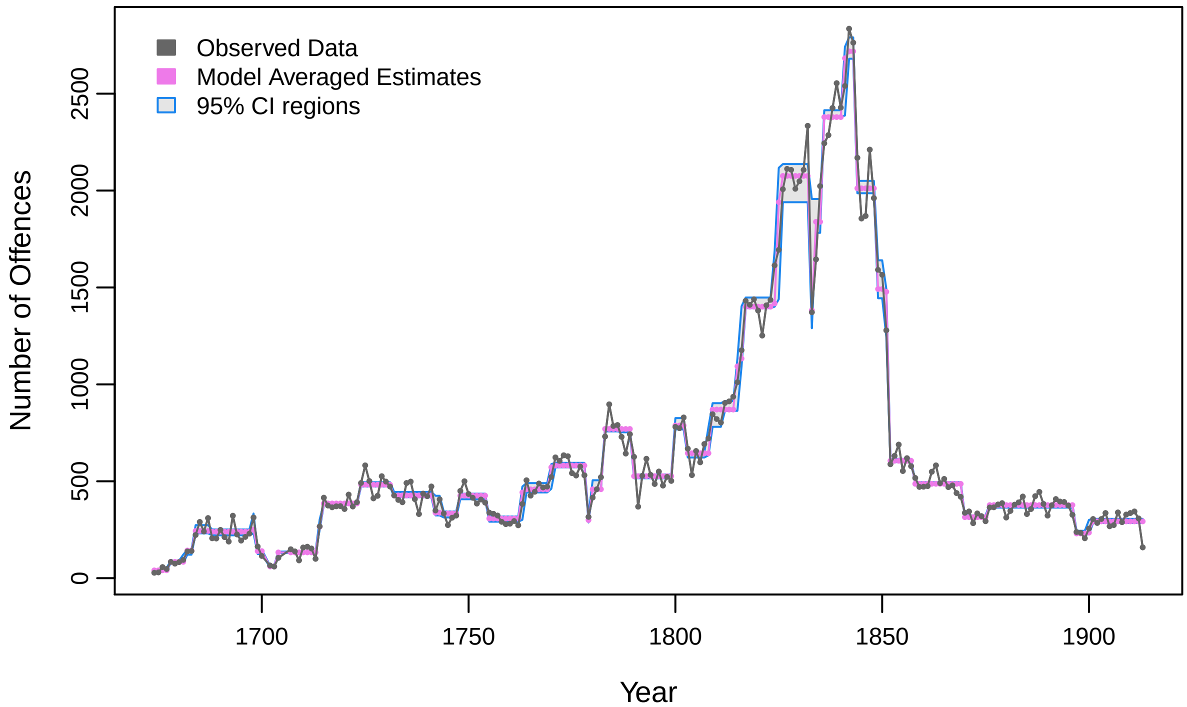


Figure 26: Model Averaged Estimates per Year, Crimes by Offence, Theft (Old Bailey Online 2018a).

k	Proportion
34	0.060
35	0.164
36	0.068
37	0.024
38	0.284
39	0.290
40	0.100
41	0.010

Table 33: Posterior estimate for k

h	Posterior Estimate	95% CI
0	38.6	(28.5, 42)
1	80.7	(50.3, 86.1)
2	142.1	(85.2, 146.4)
3	240.3	(143.3, 251.2)
4	142.3	(123.3, 314.9)
5	59.8	(55.2, 142.7)
6	133.1	(59.8, 136.5)
7	270	(61.9, 303.5)
8	388.2	(130, 388.2)
9	481.9	(251.8, 487)
10	425	(379.4, 480)
11	329.6	(310.8, 444.7)
12	425.7	(329.6, 434.3)
13	312.8	(291.4, 434.3)
14	450.9	(289.8, 490.6)
15	575.3	(295.3, 595.1)
16	305.3	(292.8, 572.1)
17	469.6	(287.3, 505.3)
18	769.9	(303, 769.9)
19	528	(458.8, 769.9)
20	787.4	(526.4, 804.6)
21	642.5	(526.4, 804.6)
22	870	(648.5, 888.7)
23	1092.7	(635, 1133.4)
24	1393.4	(863.3, 1439.5)
25	1575.6	(1400.8, 2151.3)
26	2128.9	(1382.5, 2130)
27	1369.4	(1315.5, 1838.7)
28	1838.7	(1794.2, 2379.2)
29	2392.4	(2379.2, 2722.8)
30	2718.1	(2011.5, 2742)
31	2011.5	(1573.4, 2037.2)
32	1477.6	(1243.7, 1520.9)
33	605.8	(605.8, 615.5)
34	488	(485.6, 495.1)
35	314.7	(314.4, 323.6)
36	379.5	(364.2, 381.7)
37	235.8	(221.4, 241.1)
38	300.7	(291.3, 306)

Table 34: Posterior Estimates for h, conditioned on k = 38

s	Posterior Estimate	95% CI	Year
1	4.6	(2, 5)	1679
2	8	(4.8, 8.9)	1682
3	10.6	(8.2, 10.9)	1685
4	25.3	(10.9, 25.6)	1700
5	28.4	(24.3, 28.7)	1703
6	30.9	(25.2, 31)	1705
7	40.4	(28.9, 41)	1715
8	41.5	(30.5, 41.9)	1716
9	50.2	(40.3, 51)	1725
10	58.2	(41.6, 59)	1733
11	69	(50.1, 70.4)	1743
12	74.4	(58.9, 74.8)	1749
13	81.4	(68.2, 81.9)	1756
14	89.6	(74.5, 90.7)	1764
15	96.8	(81.8, 97.6)	1771
16	105.1	(89.3, 105.8)	1780
17	106.5	(96, 106.9)	1781
18	109.4	(105.7, 109.9)	1784
19	116.8	(106.9, 116.9)	1791
20	126.4	(109.2, 126.8)	1801
21	129.5	(116.1, 129.8)	1804
22	135.2	(126.8, 136)	1810
23	141.3	(129.3, 141.9)	1816
24	143.2	(135.2, 143.8)	1818
25	150.4	(142.9, 152.2)	1825
26	152.2	(151.2, 159)	1827
27	159.2	(159, 160.6)	1834
28	160.8	(160.2, 162.4)	1835
29	162.4	(162.4, 167.1)	1837
30	167.1	(167.1, 170.9)	1842
31	170.5	(170.2, 175.9)	1845
32	175.5	(175, 177.3)	1850
33	178.5	(178.2, 178.9)	1853
34	184.7	(184.2, 184.9)	1859
35	196.3	(196.1, 196.6)	1871
36	202.6	(202.1, 202.8)	1877
37	223.5	(223.1, 224)	1898
38	227.2	(226.8, 227.6)	1902

Table 35: Posterior Estimates for s, conditioned on k = 38

Figures and Tables - Crimes by Offence - Violent Theft - (Old Bailey Online 2018a)

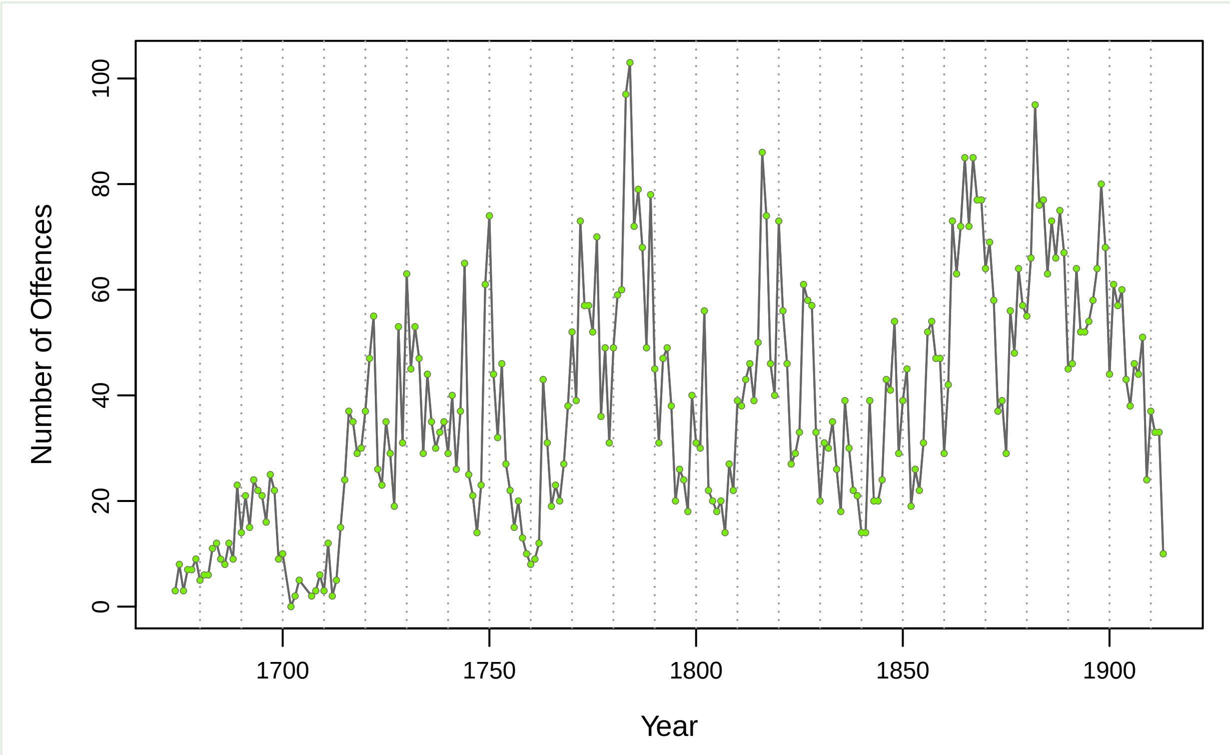


Figure 27: Number of Violent Thefts heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

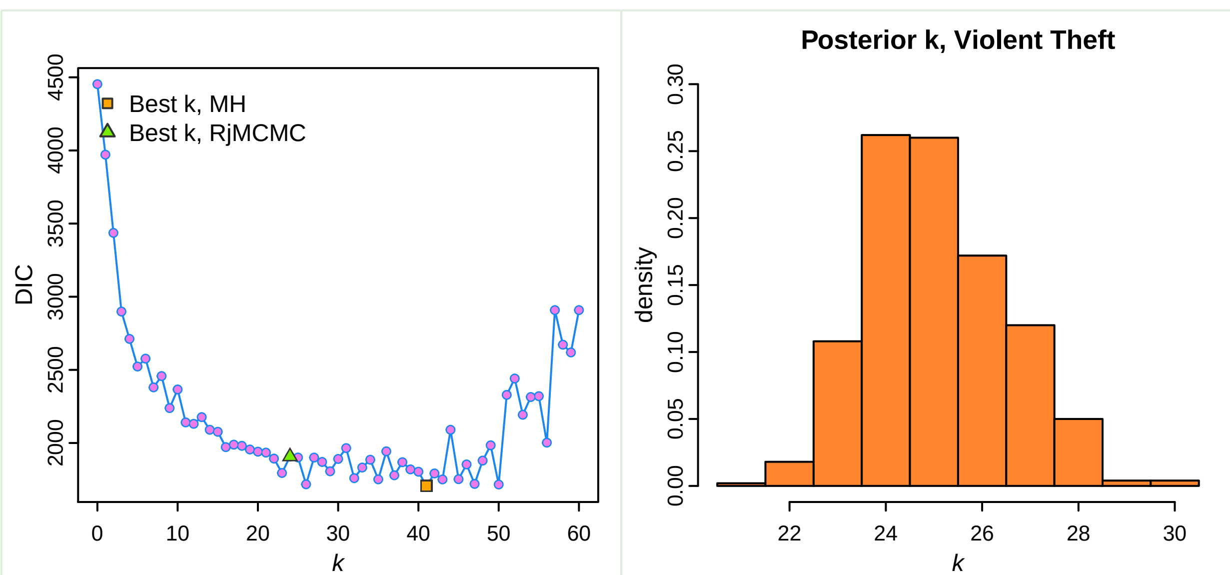


Figure 28: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Violent Theft (Old Bailey Online 2018a).

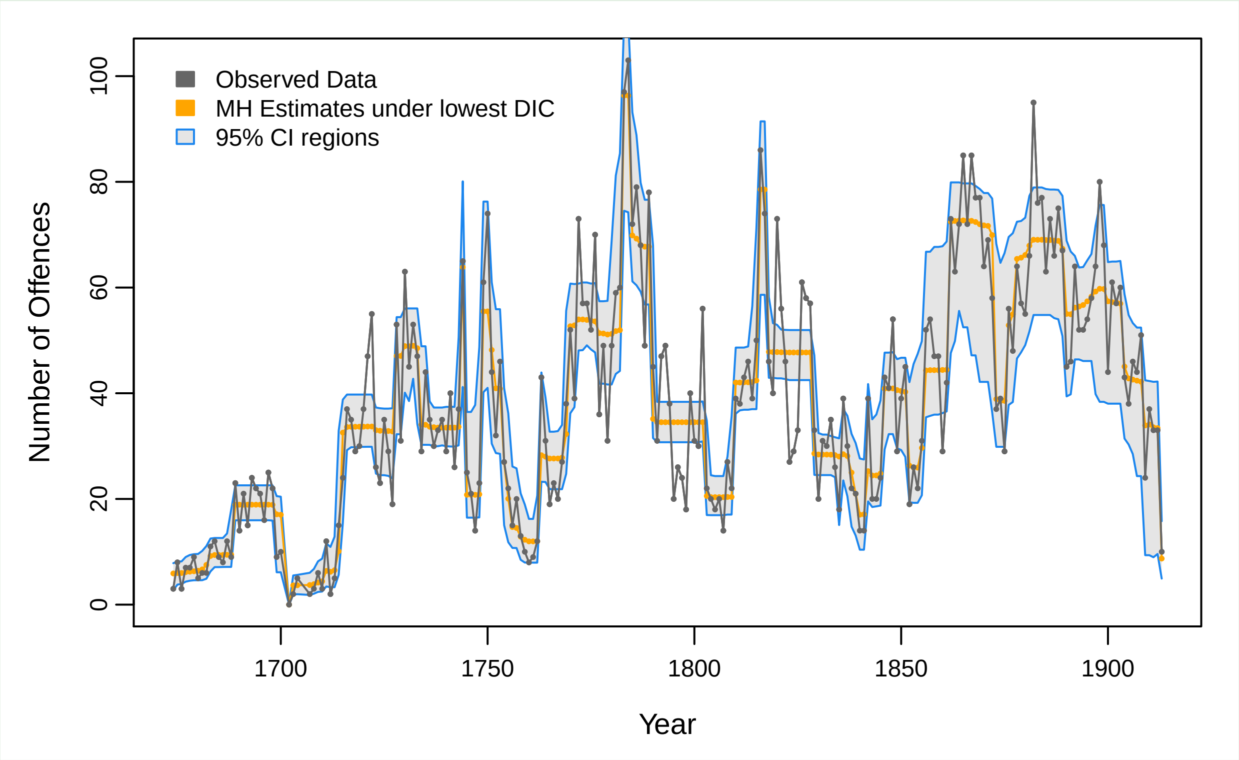


Figure 29: MH Estimates per Year, Crimes by Offence, Violent Theft (Old Bailey Online 2018a).

k	DIC
0	4453.2
1	3971.2
2	3436.4
3	2898.6
4	2711.7
5	2522.8
6	2576.8
7	2380.3
8	2457.5
9	2238.8
10	2365.9
11	2140.9
12	2131.0
13	2176.9
14	2090.4
15	2076.7
16	1971.9
17	1988.6
18	1980.5
19	1955.5
20	1940.7
21	1934.4
22	1893.4
23	1795.1
24	1907.3
25	1900.7
26	1717.6
27	1900.5
28	1870.5
29	1806.4
30	1890.9
31	1965.0
32	1759.6
33	1832.2
34	1885.7
35	1751.5
36	1943.0
37	1779.1
38	1868.7
39	1819.6
40	1803.8
41	1706.5
42	1792.0
43	1751.0
44	2090.2
45	1752.7
46	1853.6
47	1720.4
48	1879.5
49	1984.2
50	1716.3
51	2329.0
52	2440.9
53	2193.0
54	2314.4
55	2320.1
56	2002.5
57	2908.5
58	2672.0
59	2619.2
60	2908.5

Table 36: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	5.9	(2.7, 7.8)
1	9.4	(7.1, 12.6)
2	18.9	(16, 22.6)
3	0	(0, 13)
4	2.5	(0, 5.4)
5	4.9	(2.1, 22.6)
6	12.4	(4.3, 39.3)
7	33.5	(23.9, 51.1)
8	48.6	(33.4, 56)
9	33.5	(29.9, 37.4)
10	63.8	(41.6, 80.1)
11	20.8	(16.5, 36.5)
12	55.5	(41.4, 76.3)
13	28.6	(16.4, 41.4)
14	12	(7.9, 16.2)
15	28.3	(23.8, 44.1)
16	49.6	(21.8, 60.7)
17	51.2	(41.7, 58.2)
18	96.3	(74.2, 110.5)
19	67.7	(56.8, 76.6)
20	34.5	(30.7, 38.4)
21	20.3	(16.9, 24.3)
22	42.1	(36.9, 48.6)
23	78.5	(58.6, 91.4)
24	47.7	(42.5, 52)
25	28.4	(24.5, 31.9)
26	17.1	(10.4, 35)
27	25.7	(14.1, 41.4)
28	40.2	(19.5, 46.4)
29	25.9	(19.3, 47.8)
30	44.4	(36, 67.8)
31	72.6	(58, 79.9)
32	41.2	(29.9, 74.6)
33	60	(32.3, 73.7)
34	65.5	(39.8, 78.5)
35	57.9	(43.8, 75.7)
36	53.3	(36.8, 65)
37	37.9	(13.6, 48.4)
38	10.8	(5.1, 36.5)
39	6.5	(3.4, 34.9)
40	8.8	(5.1, 18.5)
41	18.3	(5.2, 22.2)

Table 37: MH Posterior Estimates for h, conditioned on k = 41

s	Posterior Estimate	95% CI	Year
1	8.8	(1.7, 10.7)	1683
2	15.5	(14.2, 16)	1690
3	27.2	(25.1, 28.9)	1702
4	29.1	(27.2, 30)	1704
5	34.4	(29.1, 40.8)	1709
6	40	(34.8, 42.8)	1714
7	42	(40.8, 54.8)	1716
8	54.9	(54, 60.7)	1729
9	60.6	(59.8, 65.2)	1735
10	70.4	(69.5, 71)	1745
11	71.5	(71, 72)	1746
12	75.5	(74.8, 76.7)	1750
13	78.9	(77.1, 81)	1753
14	82.8	(81, 86.8)	1757
15	89.5	(89, 90)	1764
16	95.9	(90.6, 97)	1770
17	99.9	(95.7, 105.8)	1774
18	109.4	(107.2, 110)	1784
19	111.6	(111, 114.1)	1786
20	116.6	(116, 117.7)	1791
21	129.6	(129, 130.4)	1804
22	136.5	(136, 137)	1811
23	142.3	(141.3, 143)	1817
24	144.5	(144, 145.5)	1819
25	155.5	(155, 156.7)	1830
26	164.9	(161, 167.1)	1839
27	168.4	(162.5, 170.6)	1843
28	172.3	(164.5, 173)	1847
29	178.4	(172, 179.9)	1853
30	182.2	(175.6, 183.1)	1857
31	188.5	(187.7, 190.4)	1863
32	199.1	(189.7, 200.2)	1874
33	202.6	(198.4, 207.8)	1877
34	208.2	(202.1, 216.9)	1883
35	216.9	(214.3, 224.3)	1891
36	226.2	(218.1, 232.2)	1901
37	231.6	(230.1, 236)	1906
38	239.1	(235.1, 239.6)	1913
39	239.5	(236.5, 240.3)	1913
40	240	(239.1, 240.7)	1913
41	240.6	(239.8, 240.9)	1913

Table 38: MH Posterior Estimates for s, conditioned on k = 41

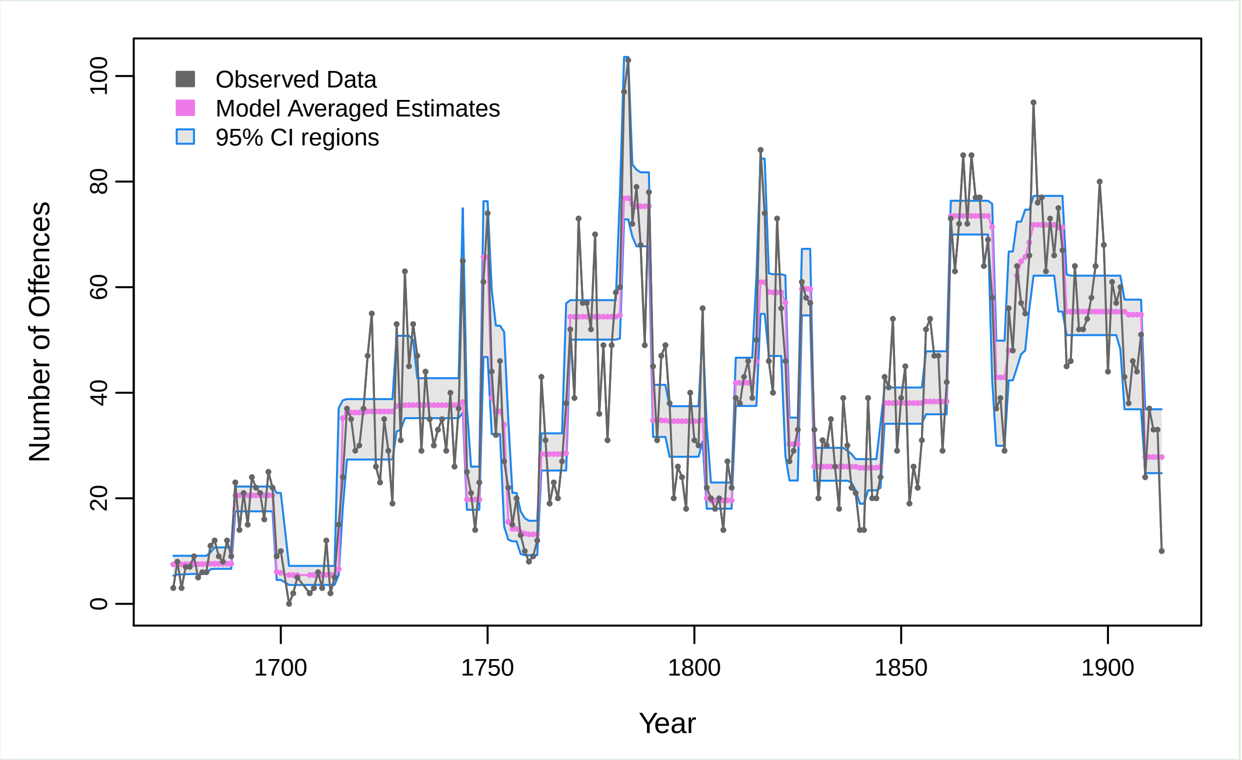


Figure 30: Model Averaged Estimates per Year, Crimes by Offence, Violent Theft (Old Bailey Online 2018a).

k	Proportion
21	0.002
22	0.018
23	0.108
24	0.262
25	0.260
26	0.172
27	0.120
28	0.050
29	0.004
30	0.004

Table 39: Posterior estimate for k

h	Posterior Estimate	95% CI
0	7.6	(5, 9.2)
1	20.6	(8.6, 22.5)
2	5.9	(3.7, 20.7)
3	36.2	(5.5, 38.1)
4	23.5	(18, 63.8)
5	50.8	(18, 76.1)
6	34.9	(14.2, 78.9)
7	21	(10.2, 49.8)
8	26.3	(9.9, 54.4)
9	52.9	(25.3, 99.5)
10	71.4	(51.2, 80.1)
11	36.1	(32.9, 82.5)
12	22.7	(18.5, 40.4)
13	39.8	(18.5, 45.8)
14	58.6	(38.3, 84.3)
15	51.7	(24.8, 62.2)
16	32.1	(24.2, 65)
17	59	(23.9, 67.3)
18	26	(22.2, 39.2)
19	38.4	(35.9, 74.4)
20	71.6	(29.9, 76.1)
21	46	(29.9, 73.3)
22	71.3	(55.6, 77.3)
23	55.4	(41.8, 57.6)
24	26.7	(24.8, 32)

Table 40: Posterior Estimates for h, conditioned on k = 24

s	Posterior Estimate	95% CI	Year
1	15.3	(5.1, 15.9)	1690
2	25.8	(15.1, 28.1)	1700
3	41.1	(25.8, 41.6)	1716
4	71.1	(40.3, 72)	1746
5	75.2	(54.6, 75.8)	1750
6	77.3	(71.4, 82)	1752
7	81.1	(75.7, 89)	1756
8	89.7	(80.8, 96.9)	1764
9	95.5	(89.1, 109.6)	1770
10	109.1	(95.7, 111.6)	1784
11	116.2	(109.1, 116.9)	1791
12	129.1	(116.2, 129.9)	1804
13	136.5	(129.1, 137)	1811
14	141.3	(136.2, 142.9)	1816
15	144.3	(141.2, 149.9)	1819
16	149.6	(148, 152.9)	1824
17	152.8	(152.1, 155.8)	1827
18	155.6	(155.3, 172.9)	1830
19	172.7	(172, 188.5)	1847
20	188.5	(188, 199.7)	1863
21	199.3	(198.2, 207.2)	1874
22	205.9	(202.1, 216.4)	1880
23	216.4	(214.3, 230.1)	1891
24	235.4	(235, 235.9)	1910

Table 41: Posterior Estimates for s, conditioned on k = 24

Figures and Tables - Crimes by Offence - Breaking the Peace - (Old Bailey Online 2018a)

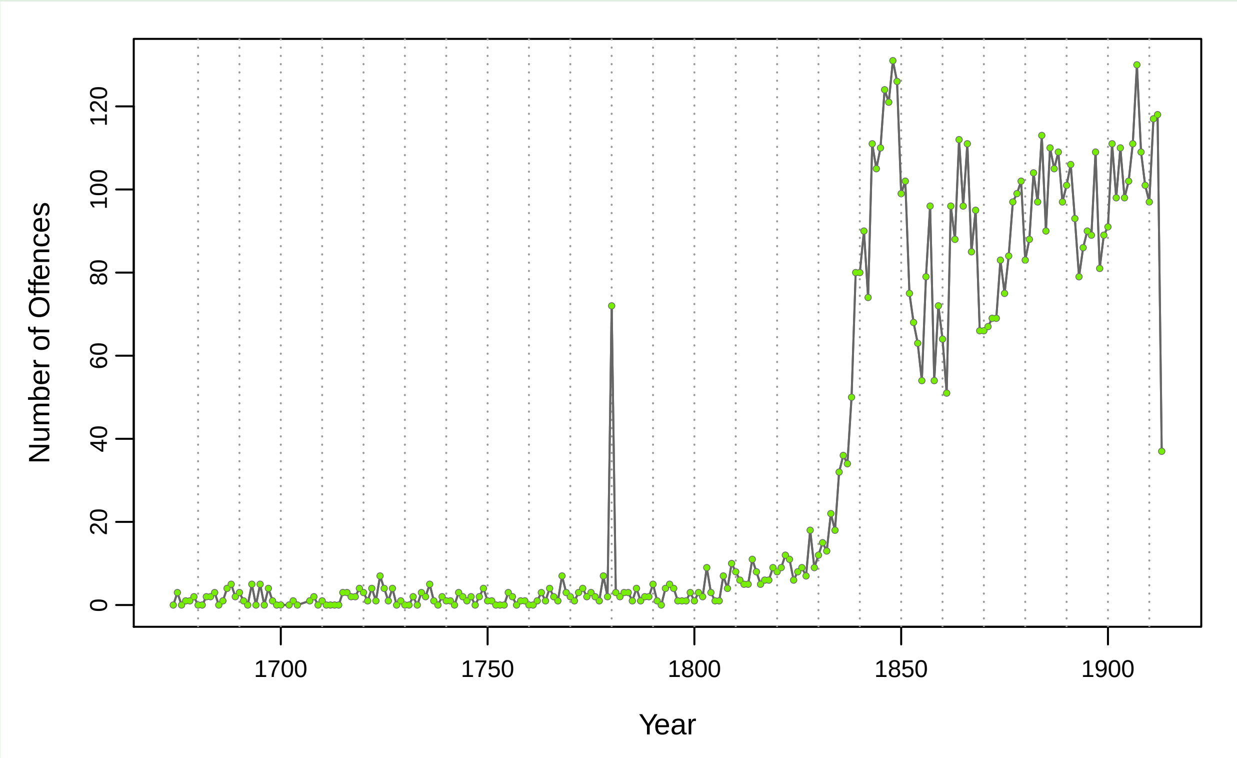
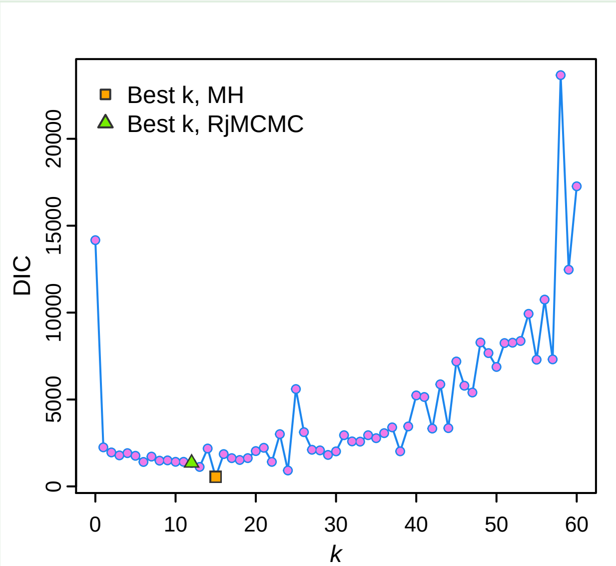


Figure 31: Number of Offences of Breaking the Peace heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).



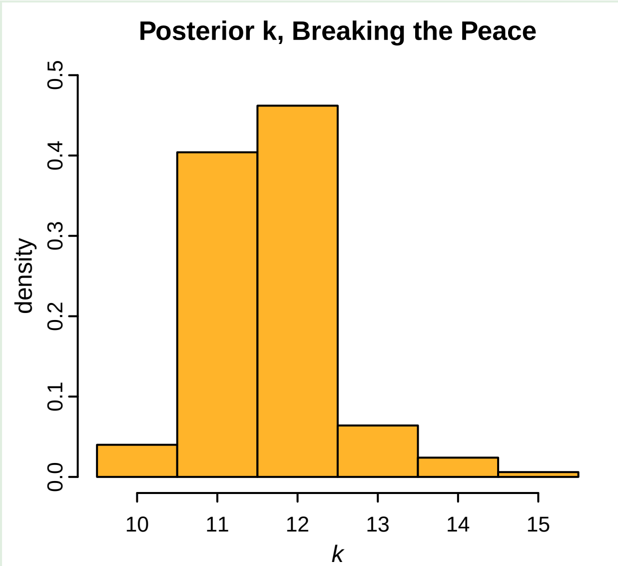


Figure 32: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Breaking the Peace (Old Bailey Online 2018a).

Figures and Tables - Crimes by Offence - Breaking the Peace - MH - (Old Bailey Online 2018a)

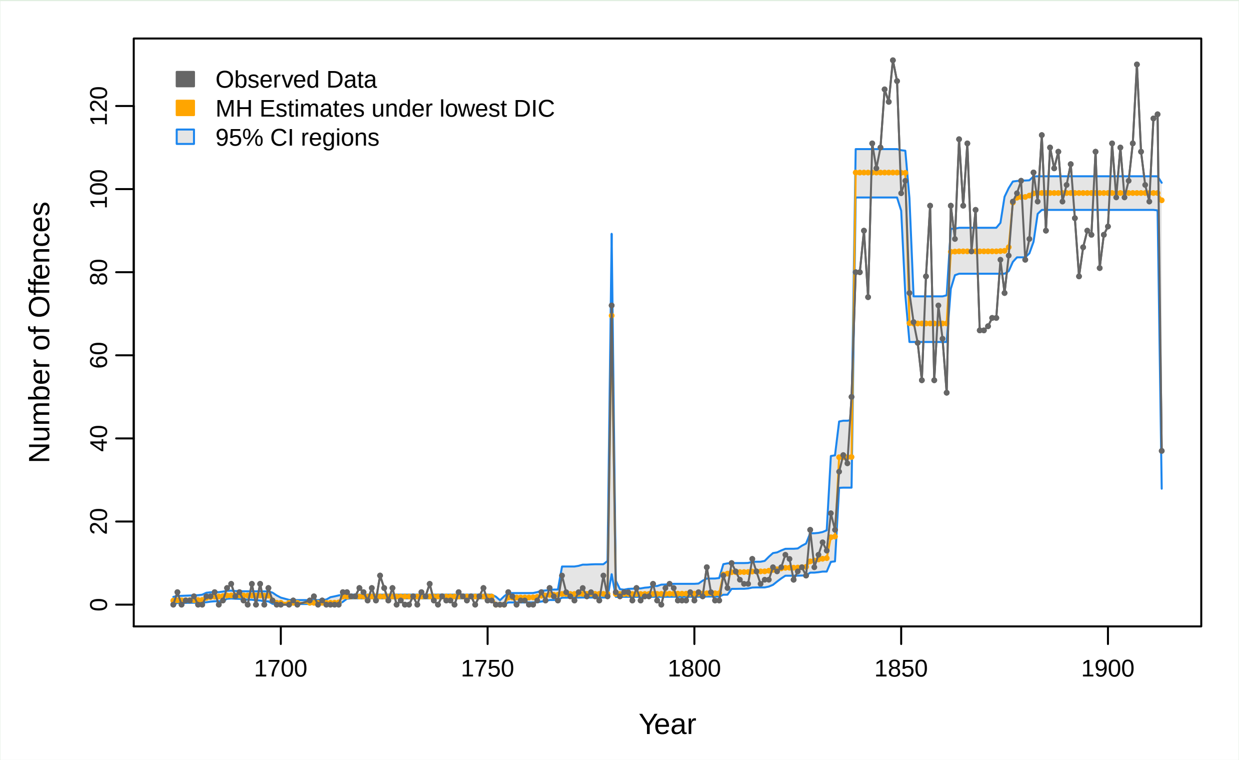


Figure 33: MH Estimates per Year, Crimes by Offence, Breaking the Peace (Old Bailey Online 2018a).

k	DIC
0	14167.4
1	2248.3
2	1954.8
3	1786.4
4	1913.8
5	1761.8
6	1406.8
7	1715.9
8	1480.6
9	1497.7
10	1414.2
11	1414.4
12	1349.5
13	1120.3
14	2179.8
15	544.9
16	1858.9
17	1627.0
18	1520.5
19	1627.1
20	2032.2
21	2221.9
22	1412.1
23	3007.4
24	916.3
25	5601.3
26	3120.0
27	2108.7
28	2073.1
29	1811.7
30	2015.3
31	2945.8
32	2589.9
33	2575.9
34	2946.1
35	2774.7
36	3064.0
37	3396.5
38	2019.0
39	3449.8
40	5234.4
41	5144.3
42	3328.1
43	5874.5
44	3353.5
45	7186.8
46	5793.4
47	5402.2
48	8280.3
49	7670.2
50	6874.2
51	8246.5
52	8267.6
53	8366.4
54	9928.7
55	7291.2
56	10748.2
57	7309.3
58	23661.1
59	12468.4
60	17266.6

Table 42: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	1	(0, 1.9)
1	2.2	(1.5, 3.4)
2	0.4	(0.2, 1.1)
3	2	(1.5, 2.4)
4	0	(0, 0.3)
5	1.7	(0.5, 2.8)
6	9.3	(2.2, 86.7)
7	2.8	(1.9, 84.6)
8	4.8	(2, 9)
9	11.2	(8, 17.9)
10	35.5	(28.2, 44.3)
11	104	(98, 109.6)
12	67.7	(63.2, 74.2)
13	85	(79.6, 90.7)
14	99.1	(95, 103.1)
15	1.3	(0.3, 46.8)

Table 43: MH Posterior Estimates for h, conditioned on k = 15

s	Posterior Estimate	95% CI	Year
1	9.2	(1.5, 14.8)	1684
2	24.9	(18.3, 27.5)	1699
3	41.3	(38.4, 42.6)	1716
4	78.5	(77.8, 79.5)	1753
5	81.4	(80, 82)	1756
6	94.7	(88.5, 106.9)	1769
7	107.3	(106, 108)	1782
8	120.5	(107.1, 135.2)	1795
9	145.5	(133.1, 157.5)	1820
10	161.1	(159, 161.9)	1836
11	165.5	(165, 166)	1840
12	178.5	(178, 179.2)	1853
13	188.5	(188, 189.3)	1863
14	203.9	(201.4, 209.6)	1878
15	240.1	(239.1, 240.8)	1913

Table 44: MH Posterior Estimates for s, conditioned on k = 15

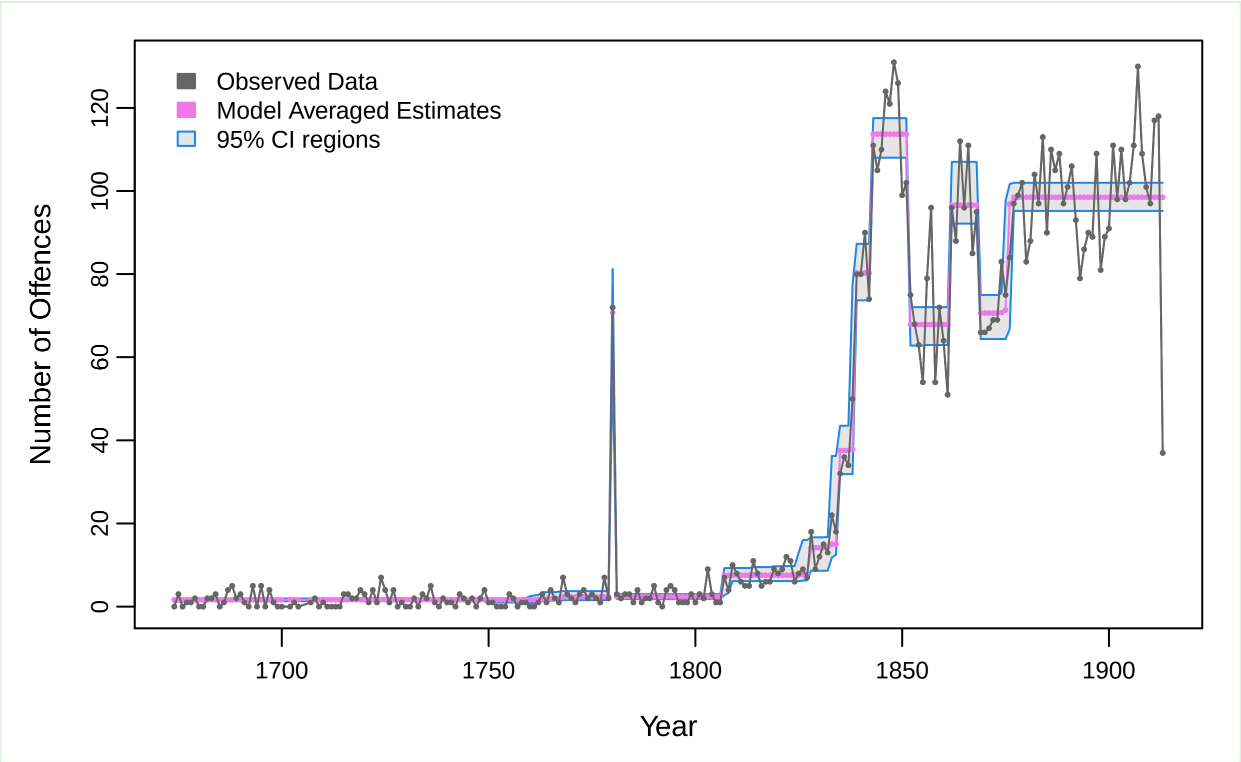


Figure 34: Model Averaged Estimates per Year, Crimes by Offence, Breaking the Peace (Old Bailey Online 2018a).

k	Proportion
10	0.040
11	0.404
12	0.462
13	0.064
14	0.024
15	0.006

Table 45: Posterior estimate for k

h	Posterior Estimate	95% CI
0	1.6	(1.1, 1.9)
1	2.8	(1.7, 4)
2	70.7	(42.3, 81)
3	2.3	(1.8, 4)
4	7.5	(6.1, 9.3)
5	14.6	(10.8, 16.8)
6	39.9	(32.1, 43.8)
7	80.2	(74.9, 86.7)
8	113.3	(107.3, 117.6)
9	67.9	(62.9, 76)
10	98.2	(92.6, 107)
11	70.3	(64.4, 74.2)
12	98.5	(95.1, 101.8)

Table 46: Posterior Estimates for h, conditioned on k = 12

s	Posterior Estimate	95% CI	Year
1	91.9	(22.5, 98.6)	1766
2	106.3	(106, 107)	1781
3	107.6	(107.1, 108)	1782
4	133.4	(133.1, 134)	1808
5	154.2	(147.5, 158.2)	1829
6	161.5	(159.7, 162)	1836
7	165.6	(165, 166)	1840
8	169.5	(169, 170)	1844
9	178.4	(178, 179)	1853
10	188.5	(188, 188.9)	1863
11	195.6	(195.1, 196)	1870
12	202.9	(200.8, 203.9)	1877

Table 47: Posterior Estimates for s, conditioned on k = 12

Figures and Tables - Crimes by Offence - Damage to Property - (Old Bailey Online 2018a)

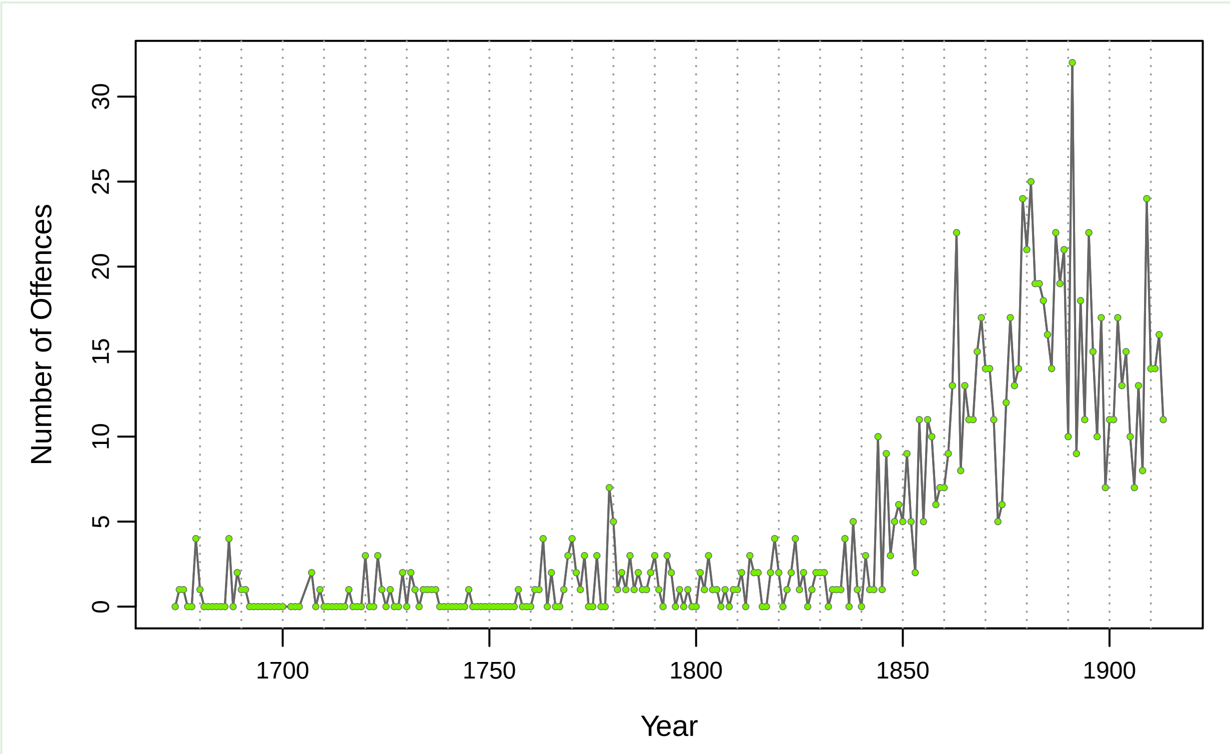


Figure 35: Number of Offences of Damage to Property heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

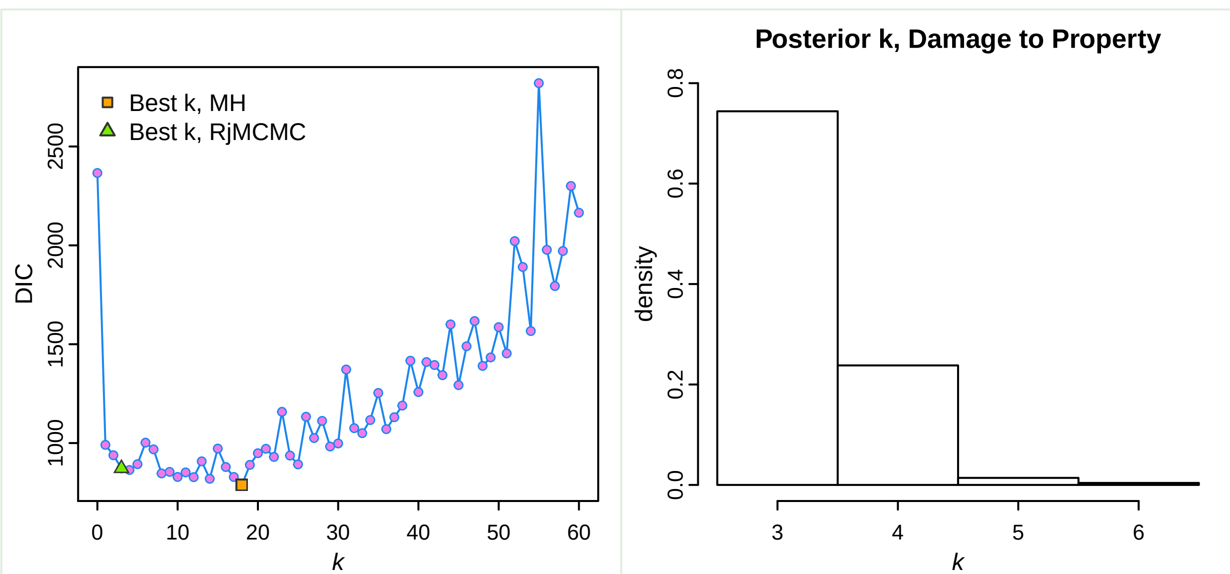


Figure 36: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Damage to Property (Old Bailey Online 2018a).

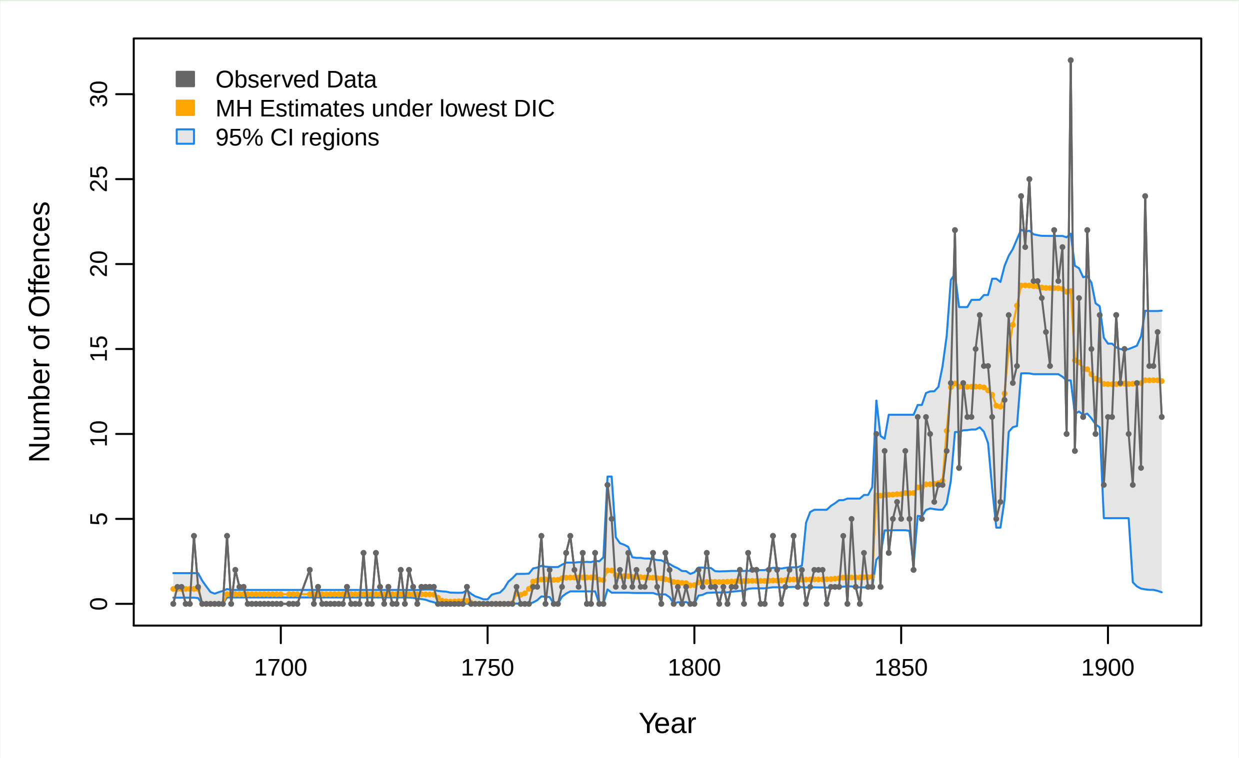


Figure 37: MH Estimates per Year, Crimes by Offence, Damage to Property (Old Bailey Online 2018a).

k	DIC
0	2366.1
1	991.0
2	938.5
3	872.1
4	863.9
5	893.3
6	1002.1
7	968.2
8	846.5
9	854.2
10	828.5
11	851.6
12	827.6
13	907.6
14	819.3
15	972.0
16	878.8
17	828.6
18	788.3
19	889.8
20	948.6
21	971.1
22	930.0
23	1158.2
24	936.9
25	892.3
26	1133.2
27	1025.7
28	1112.2
29	982.6
30	998.0
31	1372.1
32	1075.3
33	1050.2
34	1116.6
35	1253.4
36	1070.5
37	1130.8
38	1189.7
39	1416.4
40	1258.1
41	1409.7
42	1394.8
43	1343.5
44	1600.2
45	1293.1
46	1489.7
47	1617.2
48	1390.4
49	1433.2
50	1586.3
51	1453.5
52	2021.6
53	1890.7
54	1566.7
55	2820.4
56	1977.5
57	1794.2
58	1971.5
59	2300.4
60	2165.1

Table 48: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	0.9	(0.4, 1.8)
1	0	(0, 0.1)
2	0.6	(0.4, 0.8)
3	0	(0, 0.6)
4	0	(0, 1.7)
5	0	(0, 1.3)
6	1.2	(0, 2.4)
7	1.5	(0, 5.2)
8	1.2	(0, 7.2)
9	1.3	(0.5, 2.2)
10	1.5	(0.9, 3)
11	5.9	(1.2, 8)
12	9.7	(2.3, 16)
13	10.1	(4.5, 18.3)
14	16.5	(10.2, 21.1)
15	14.5	(8, 21.7)
16	12.9	(0.4, 21.2)
17	1.3	(0, 20.1)
18	0.6	(0, 15.6)

Table 49: MH Posterior Estimates for h, conditioned on k = 18

s	Posterior Estimate	95% CI	Year
1	7.8	(7, 10.9)	1682
2	13.1	(9.9, 14)	1688
3	64.8	(58.2, 73.6)	1739
4	74.5	(67.1, 87.4)	1749
5	80.7	(72.6, 92.6)	1755
6	86.7	(74.8, 96)	1761
7	103	(82.1, 105.8)	1777
8	114.8	(105.1, 125.9)	1789
9	124.6	(107.2, 130.3)	1799
10	148.5	(129.2, 165.8)	1823
11	170.2	(147.9, 171)	1845
12	180.9	(170, 189)	1855
13	188.9	(173.7, 199.8)	1863
14	201.1	(186.2, 205.9)	1876
15	205.8	(198.3, 223.7)	1880
16	222.1	(202.5, 240.3)	1897
17	240	(212.5, 240.7)	1913
18	240.5	(218.6, 240.9)	1913

Table 50: MH Posterior Estimates for s, conditioned on k = 18

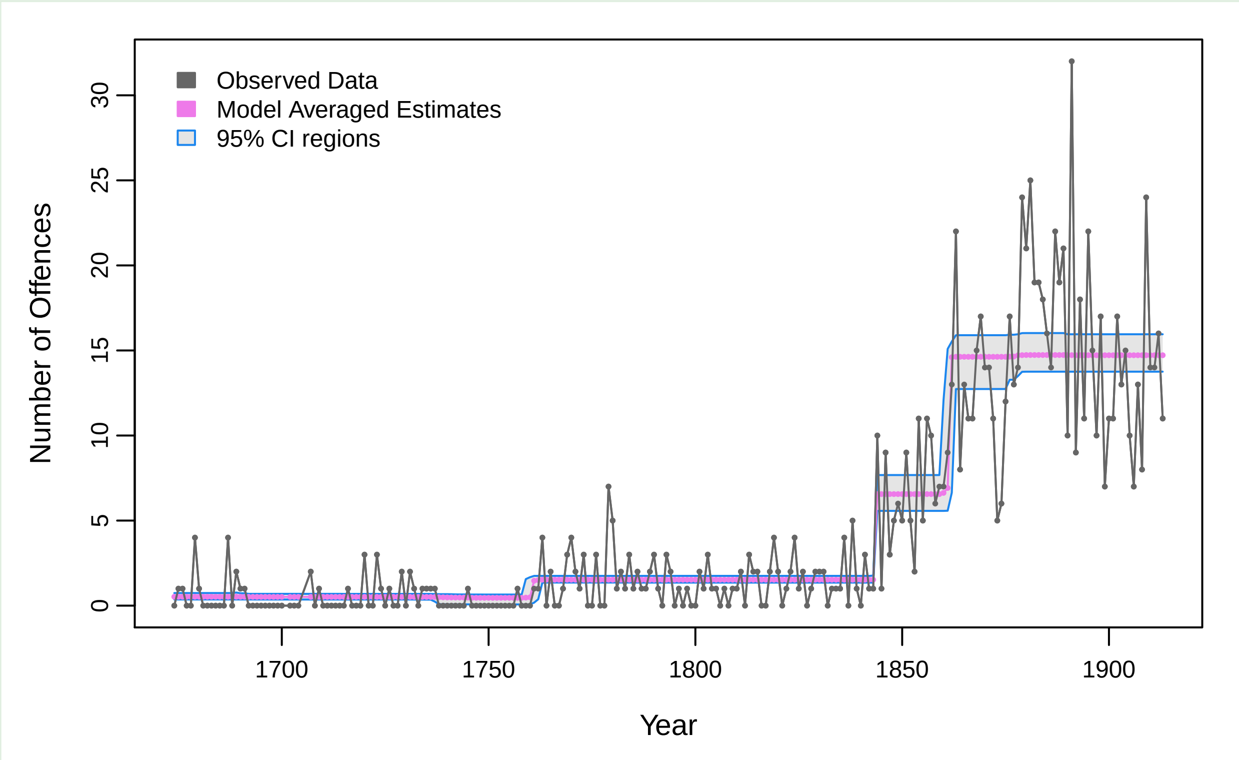


Figure 38: Model Averaged Estimates per Year, Crimes by Offence, Damage to Property (Old Bailey Online 2018a).

k	Proportion
3	0.744
4	0.238
5	0.014
6	0.004

Table 51: Posterior estimate for k

h	Posterior Estimate	95% CI
0	0.5	(0.4, 0.7)
1	1.5	(1.4, 1.7)
2	6.5	(5.6, 7.7)
3	14.7	(13.8, 15.9)

Table 52: Posterior Estimates for h, conditioned on k = 3

s	Posterior Estimate	95% CI	Year
1	87.8	(85.2, 89.8)	1762
2	170.6	(170, 171)	1845
3	188.4	(187, 189.6)	1863

Table 53: Posterior Estimates for s, conditioned on k = 3

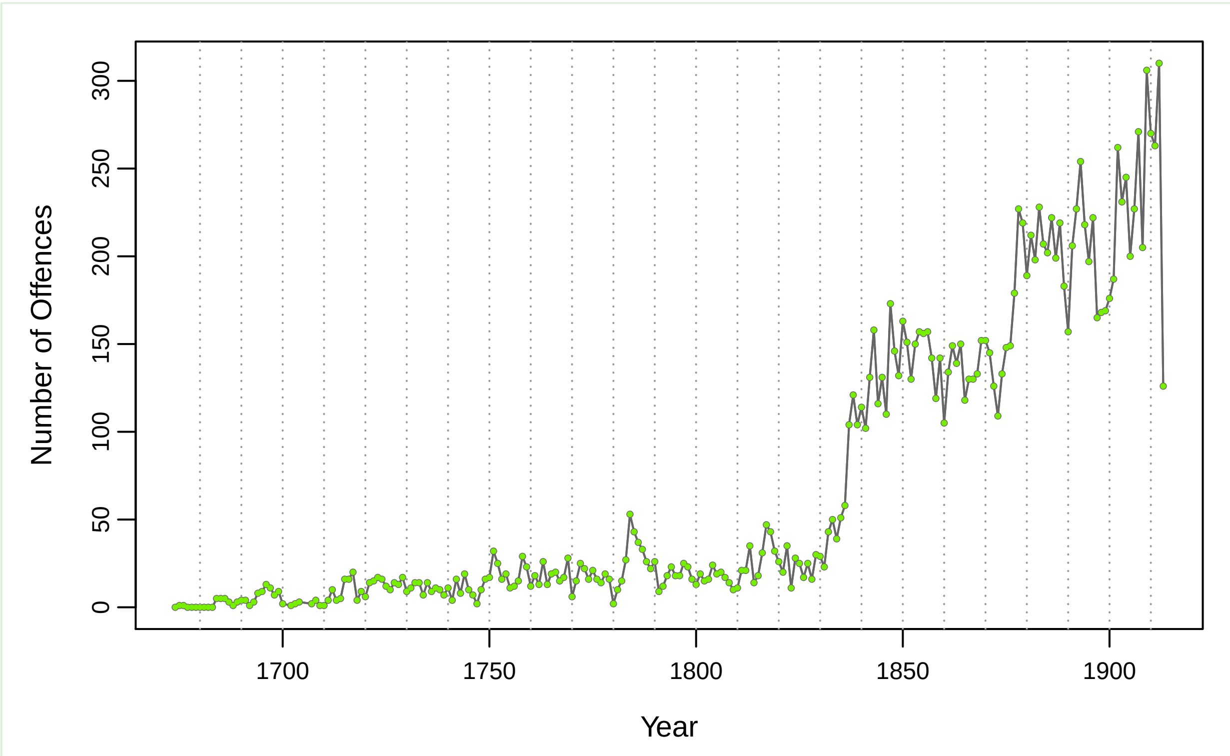


Figure 39: Number of Offences of Deception heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

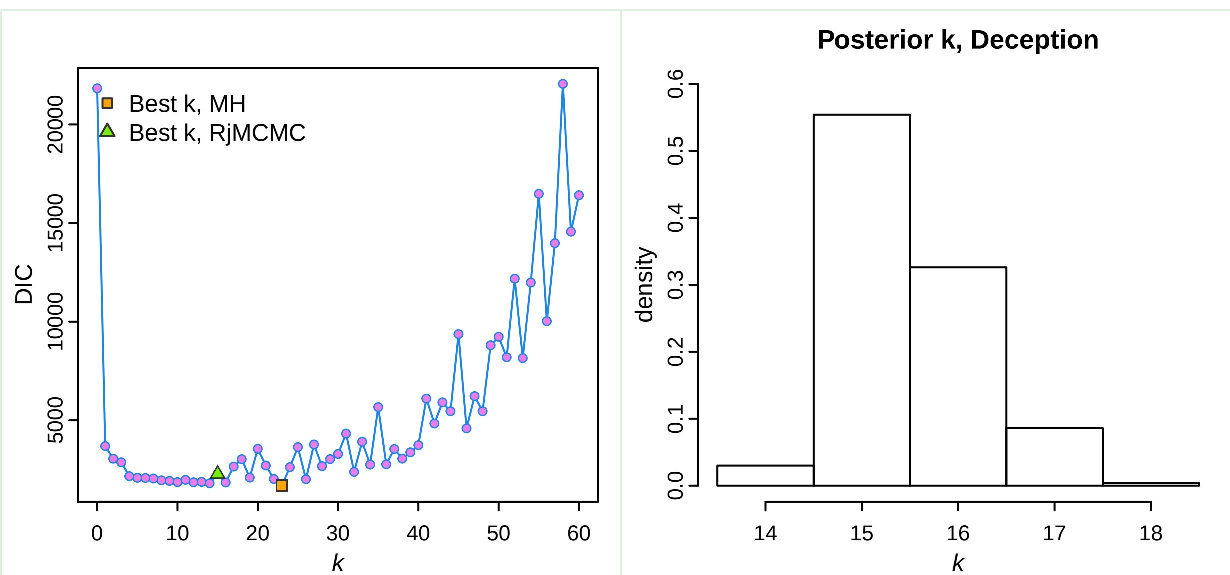


Figure 40: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Deception (Old Bailey Online 2018a).

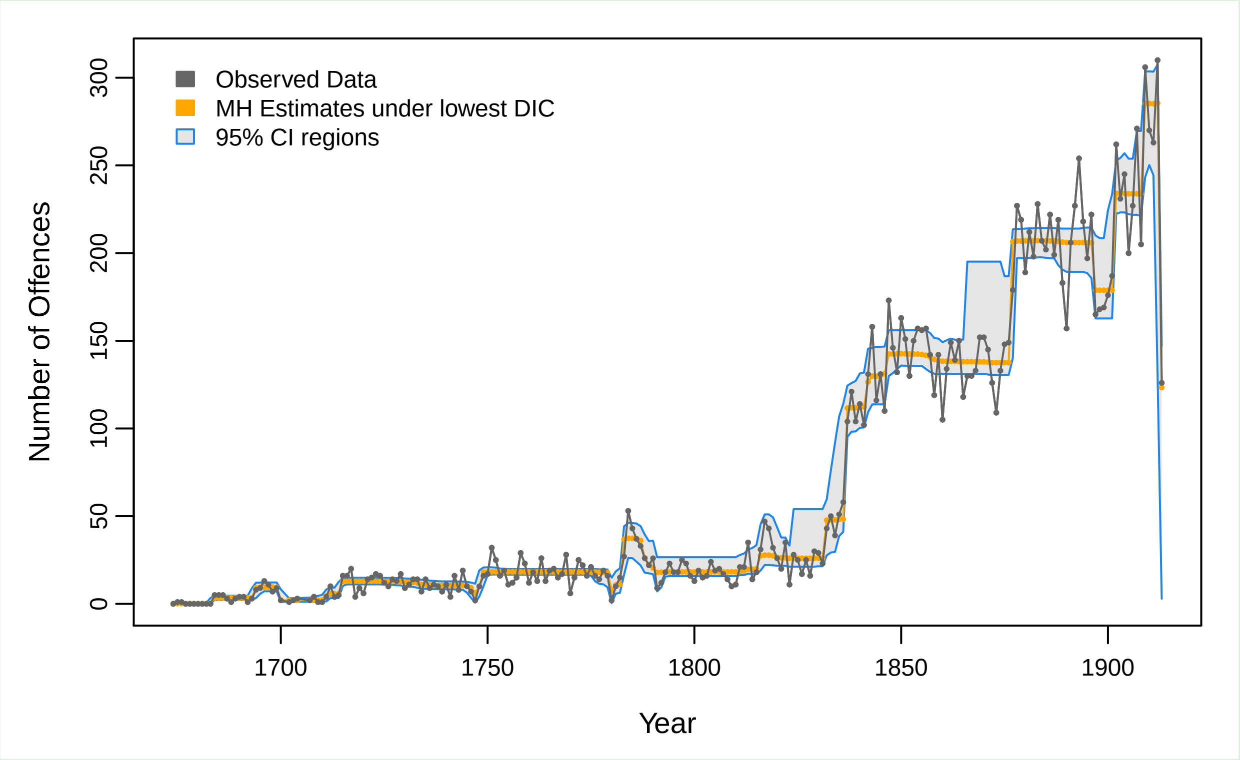


Figure 41: MH Estimates per Year, Crimes by Offence, Deception (Old Bailey Online 2018a).

k	DIC
0	21832.6
1	3691.6
2	3053.4
3	2869.1
4	2165.1
5	2084.3
6	2075.5
7	2047.1
8	1955.9
9	1926.3
10	1864.2
11	1980.7
12	1854.4
13	1876.9
14	1801.2
15	2275.8
16	1847.3
17	2654.3
18	3026.6
19	2095.7
20	3554.7
21	2705.9
22	2024.1
23	1684.9
24	2622.8
25	3639.3
26	2013.1
27	3773.5
28	2673.6
29	3030.5
30	3294.3
31	4326.1
32	2384.4
33	3914.0
34	2754.8
35	5665.5
36	2769.3
37	3543.9
38	3057.5
39	3376.0
40	3737.7
41	6098.8
42	4836.5
43	5908.9
44	5458.0
45	9369.3
46	4589.3
47	6221.9
48	5456.5
49	8803.1
50	9235.5
51	8194.5
52	12179.2
53	8154.7
54	11987.1
55	16481.8
56	10025.2
57	13980.7
58	22057.0
59	14565.1
60	16412.6

Table 54: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	0.2	(0, 0.6)
1	3.4	(2.3, 4.6)
2	9.5	(7.2, 12.2)
3	2.1	(1.2, 3.2)
4	5.7	(3.2, 9.3)
5	12.9	(11.1, 14.9)
6	10.6	(8.4, 13)
7	6.3	(0.7, 20.7)
8	18.2	(16.6, 19.8)
9	7.9	(0.4, 14.8)
10	34	(9.5, 44.3)
11	27.3	(7.1, 43.7)
12	18.2	(15.8, 26.6)
13	28	(24.1, 54.8)
14	46.5	(21.7, 114.9)
15	106.7	(44.1, 138.7)
16	130.3	(113.7, 147.6)
17	141.3	(133.2, 178.1)
18	203.2	(130.6, 213.9)
19	197.9	(163.6, 240.7)
20	231.9	(165.5, 284.9)
21	276.3	(118.3, 301.5)
22	131.3	(0.2, 304.8)
23	0.4	(0, 139.9)

Table 55: MH Posterior Estimates for h, conditioned on k = 23

s	Posterior Estimate	95% CI	Year
1	10.5	(9.1, 11)	1685
2	20.5	(19.4, 22)	1695
3	26.5	(26, 27.7)	1701
4	37.7	(33.4, 38.9)	1712
5	41.5	(40.5, 42.1)	1716
6	59.9	(51.2, 65)	1734
7	72.5	(63.7, 75.3)	1747
8	75.3	(74.1, 81.4)	1750
9	106.3	(102.3, 106.9)	1781
10	109.2	(107.1, 110)	1784
11	113.5	(109.1, 117.8)	1788
12	116.7	(113.9, 119.7)	1791
13	142.2	(137.3, 158.1)	1817
14	158.2	(145.7, 163.3)	1833
15	163.2	(158, 168.4)	1838
16	168.4	(163.1, 173.3)	1843
17	173.8	(172.1, 188.9)	1848
18	203.1	(183, 204.8)	1878
19	215.8	(203.1, 228.8)	1890
20	228.3	(215.1, 235.7)	1903
21	235.3	(228, 239.1)	1910
22	239.2	(235.1, 239.9)	1913
23	240.1	(239.1, 240.8)	1913

Table 56: MH Posterior Estimates for s, conditioned on k = 23

Figures and Tables - Crimes by Offence - Deception - RjMCMC - (Old Bailey Online 2018a)

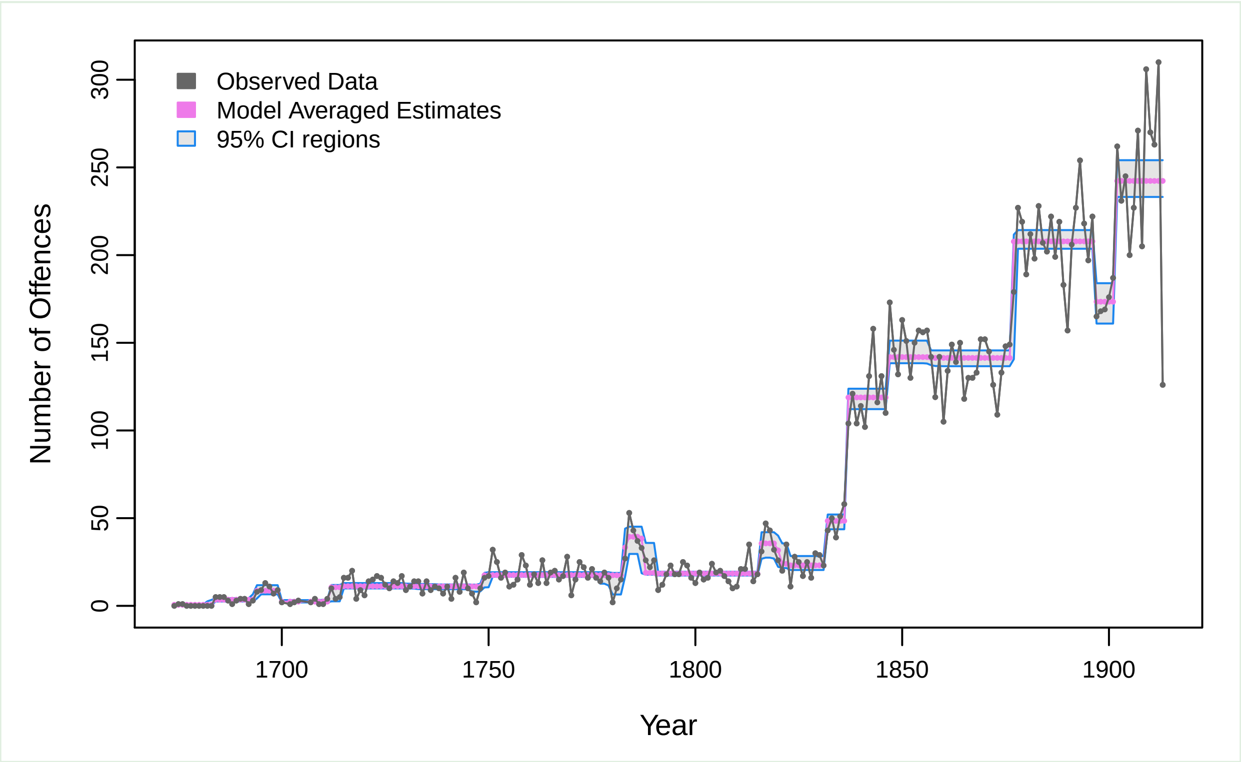


Figure 42: Model Averaged Estimates per Year, Crimes by Offence, Deception (Old Bailey Online 2018a).

k	Proportion
14	0.030
15	0.554
16	0.326
17	0.086
18	0.004

Table 57: Posterior estimate for k

h	Posterior Estimate	95% CI
0	0.6	(0.4, 0.7)
1	3.6	(2.6, 4.3)
2	8.7	(6.5, 11.8)
3	2.4	(1.9, 3.3)
4	11.2	(9.9, 12.1)
5	17.5	(16.4, 18.8)
6	38.4	(11.3, 45.1)
7	18.6	(17.3, 42.2)
8	35.6	(18, 42)
9	23.6	(20.9, 28.4)
10	48.5	(44.1, 51.4)
11	119.1	(113.3, 123.8)
12	141.4	(138, 145.7)
13	208.4	(202.5, 214.3)
14	171.2	(161, 182.9)
15	241.6	(233.2, 248.2)

Table 58: Posterior Estimates for h, conditioned on k = 15

s	Posterior Estimate	95% CI	Year
1	10.2	(8.5, 11)	1685
2	20.4	(20.1, 21.4)	1695
3	26.6	(26.1, 27)	1701
4	38.6	(38.1, 41.9)	1713
5	75.8	(74.8, 77.7)	1750
6	110.1	(106.9, 110.9)	1785
7	114.6	(109.2, 117.9)	1789
8	142.2	(114.1, 142.8)	1817
9	147.4	(142.5, 149.6)	1822
10	158.5	(158, 158.9)	1833
11	163.6	(163.1, 164)	1838
12	173.5	(173.1, 173.9)	1848
13	203.7	(203, 204.8)	1878
14	223.6	(223, 224)	1898
15	228.7	(228, 229)	1903

Table 59: Posterior Estimates for s, conditioned on k = 15

Figures and Tables - Crimes by Offence - Miscellaneous Offences - (Old Bailey Online 2018a)

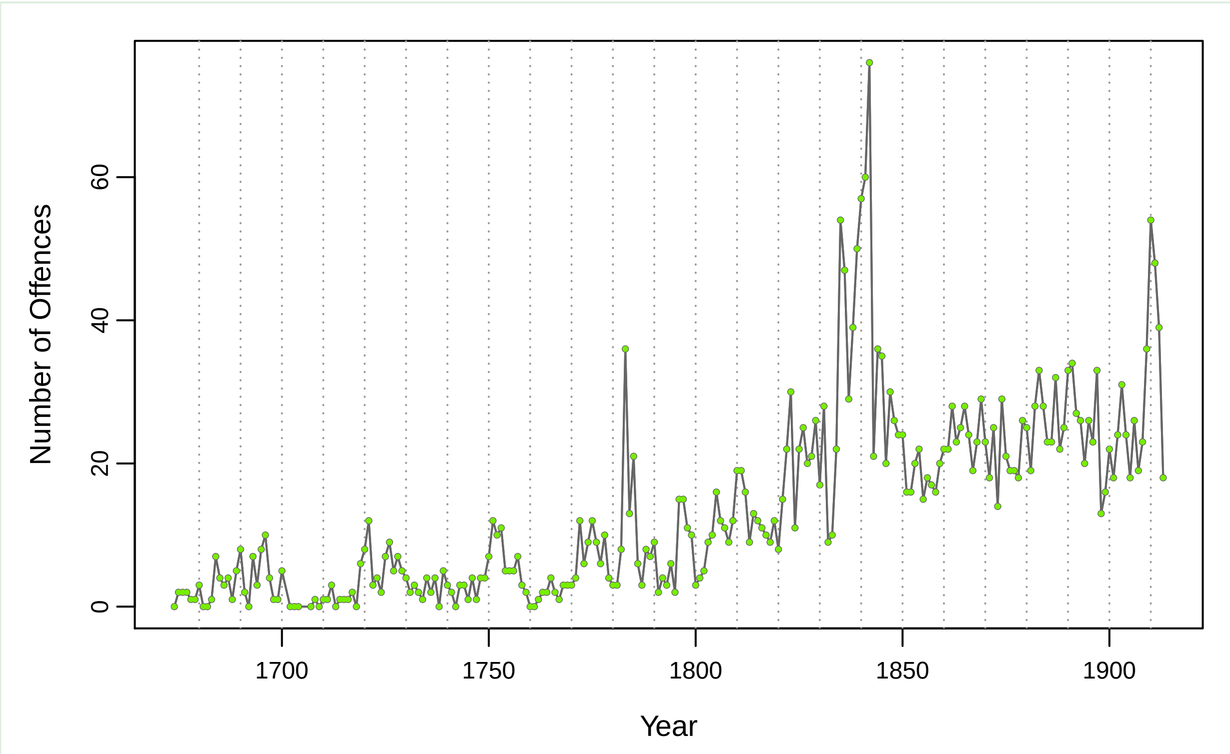


Figure 43: Number of Miscellaneous Offences heard at the Old Bailey, counting by offences per year (Old Bailey Online 2018a).

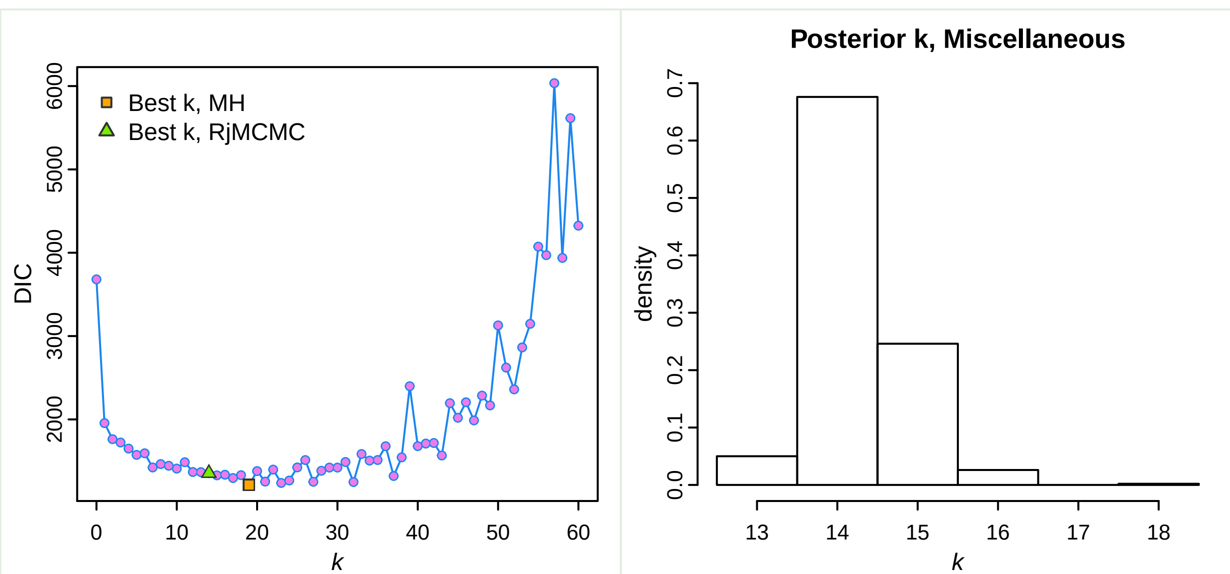


Figure 44: Model Estimates - DIC vs. Posterior k, Crimes by Offence, Miscellaneous (Old Bailey Online 2018a).

Figures and Tables - Crimes by Offence - Miscellaneous Offences - MH - (Old Bailey Online 2018a)

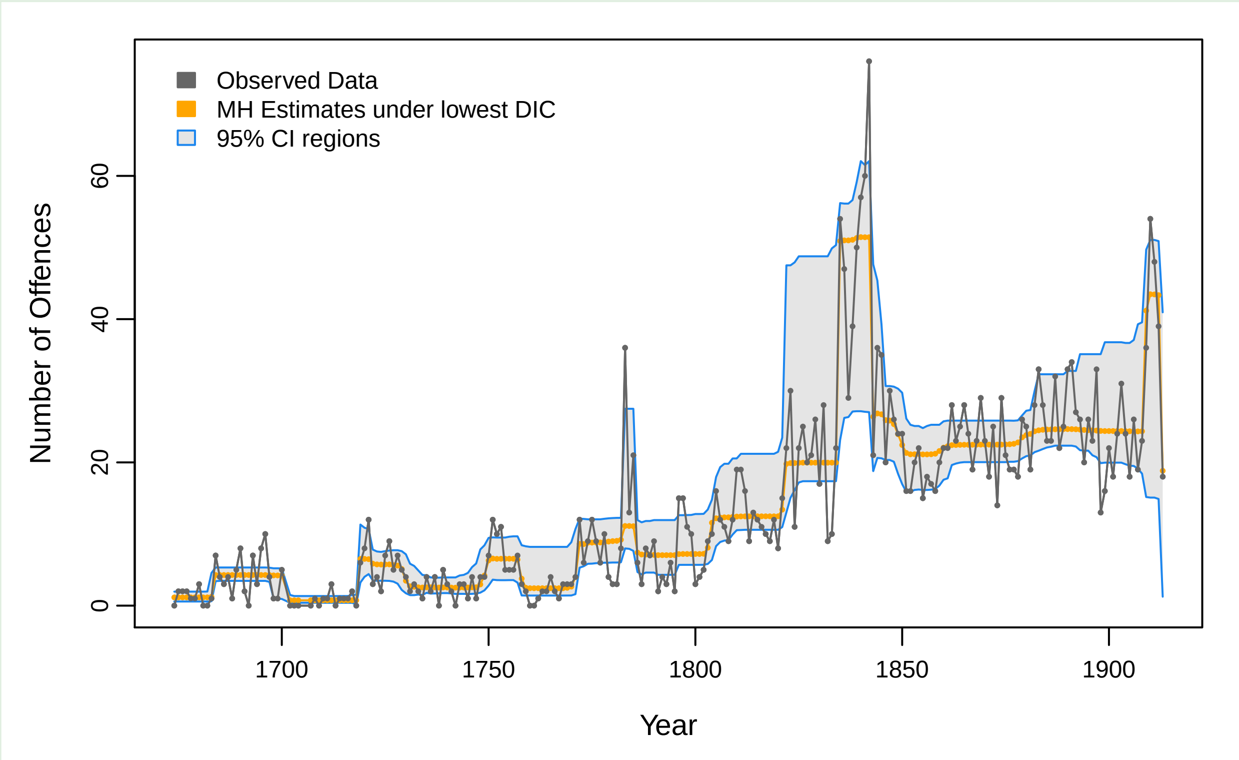


Figure 45: MH Estimates per Year, Crimes by Offence, Miscellaneous (Old Bailey Online 2018a).

k	DIC
0	3680.5
1	1955.7
2	1762.8
3	1722.7
4	1649.4
5	1574.1
6	1593.2
7	1423.0
8	1463.7
9	1443.5
10	1409.5
11	1486.1
12	1368.7
13	1367.5
14	1354.3
15	1328.7
16	1336.2
17	1296.4
18	1330.7
19	1213.5
20	1379.7
21	1253.3
22	1397.6
23	1237.3
24	1266.5
25	1423.7
26	1512.1
27	1251.2
28	1383.4
29	1421.8
30	1421.8
31	1488.9
32	1248.5
33	1584.4
34	1505.8
35	1512.7
36	1678.2
37	1320.9
38	1544.7
39	2398.5
40	1679.2
41	1709.9
42	1716.8
43	1566.8
44	2194.6
45	2019.6
46	2205.6
47	1987.9
48	2285.4
49	2167.9
50	3128.3
51	2621.7
52	2360.2
53	2864.0
54	3147.2
55	4073.2
56	3971.0
57	6035.1
58	3938.1
59	5615.3
60	4324.7

Table 60: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	1.2	(0.6, 2)
1	4.3	(3.5, 5.3)
2	0.7	(0.4, 1.3)
3	6.5	(0.2, 11.3)
4	5	(1.7, 7.7)
5	2.5	(1.3, 3.9)
6	6.3	(1.7, 9.1)
7	2.4	(1.4, 8.2)
8	8.7	(5.7, 12.1)
9	9	(5, 27.5)
10	11.3	(4.3, 18)
11	18.7	(10.9, 47.6)
12	44.1	(18.3, 55.9)
13	31	(18.7, 62.1)
14	22.4	(15.8, 36.8)
15	22.8	(20, 27.7)
16	24.4	(19.7, 36.8)
17	43.4	(14.9, 51.1)
18	16.8	(0.8, 27.2)
19	1.3	(0.6, 4.8)

Table 61: MH Posterior Estimates for h, conditioned on k = 19

s	Posterior Estimate	95% CI	Year
1	10.5	(9.2, 11)	1685
2	28	(24.4, 29)	1702
3	45.4	(44.2, 46)	1720
4	50.1	(45.7, 57.5)	1725
5	57.4	(54.4, 64.1)	1732
6	75	(58.9, 77.9)	1749
7	84.3	(73.1, 86.4)	1759
8	98.3	(95.6, 99.7)	1773
9	109.8	(101.2, 116.7)	1784
10	129.4	(112.1, 135.5)	1804
11	147.1	(122.3, 150.4)	1822
12	161.1	(147.1, 162.7)	1836
13	167.2	(161.1, 171.1)	1842
14	170	(169.1, 179.3)	1844
15	185	(172.1, 208.5)	1859
16	209.9	(204.1, 231.7)	1884
17	235.6	(233.3, 236.9)	1910
18	239.5	(239, 240.3)	1913
19	240.4	(240, 240.9)	1913

Table 62: MH Posterior Estimates for s, conditioned on k = 19

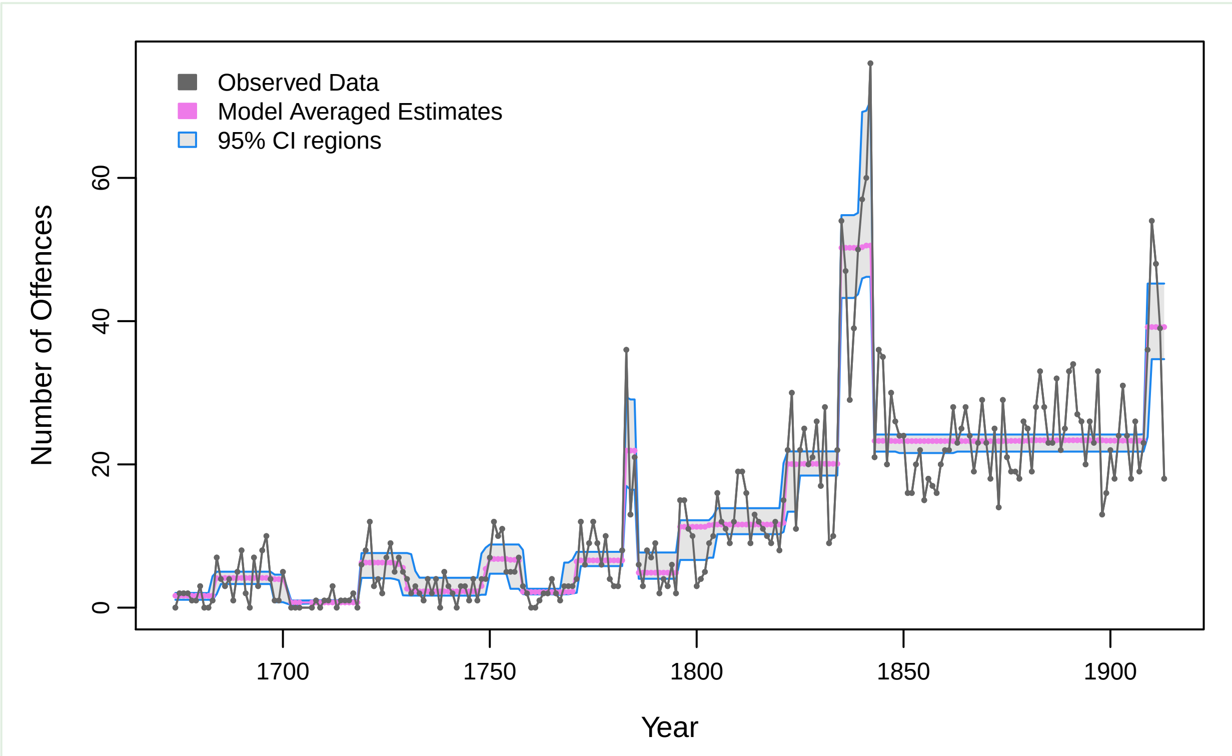


Figure 46: Model Averaged Estimates per Year, Crimes by Offence, Miscellaneous (Old Bailey Online 2018a).

k	Proportion
13	0.050
14	0.676
15	0.246
16	0.026
18	0.002

Table 63: Posterior estimate for k

h	Posterior Estimate	95% CI
0	1.6	(1.1, 2)
1	4.1	(3.3, 5)
2	0.7	(0.6, 1)
3	6.5	(5, 7.6)
4	2.2	(1.7, 3.8)
5	6.7	(4.7, 8.6)
6	2.2	(1.9, 2.7)
7	6.6	(5.8, 7.8)
8	21.7	(16.2, 29.4)
9	4.9	(4, 7.7)
10	11.6	(10.3, 13.9)
11	20.1	(18.4, 21.8)
12	50.3	(46.2, 54.8)
13	23.2	(22.4, 24.2)
14	39	(34.7, 45.2)

Table 64: Posterior Estimates for h, conditioned on k = 14

s	Posterior Estimate	95% CI	Year
1	10.5	(9.9, 11.2)	1685
2	27.8	(24.4, 28.8)	1702
3	45.7	(45, 46)	1720
4	56.4	(54.2, 59.9)	1731
5	75.6	(74.1, 76.9)	1750
6	84.4	(81.8, 86)	1759
7	97.3	(94.4, 98.9)	1772
8	109.2	(109, 109.7)	1784
9	112.4	(112, 113)	1787
10	122.5	(122, 131.9)	1797
11	148.7	(147.4, 151.9)	1823
12	161.6	(161.2, 162)	1836
13	169.5	(169, 170)	1844
14	235.4	(235, 235.9)	1910

Table 65: Posterior Estimates for s, conditioned on k = 14

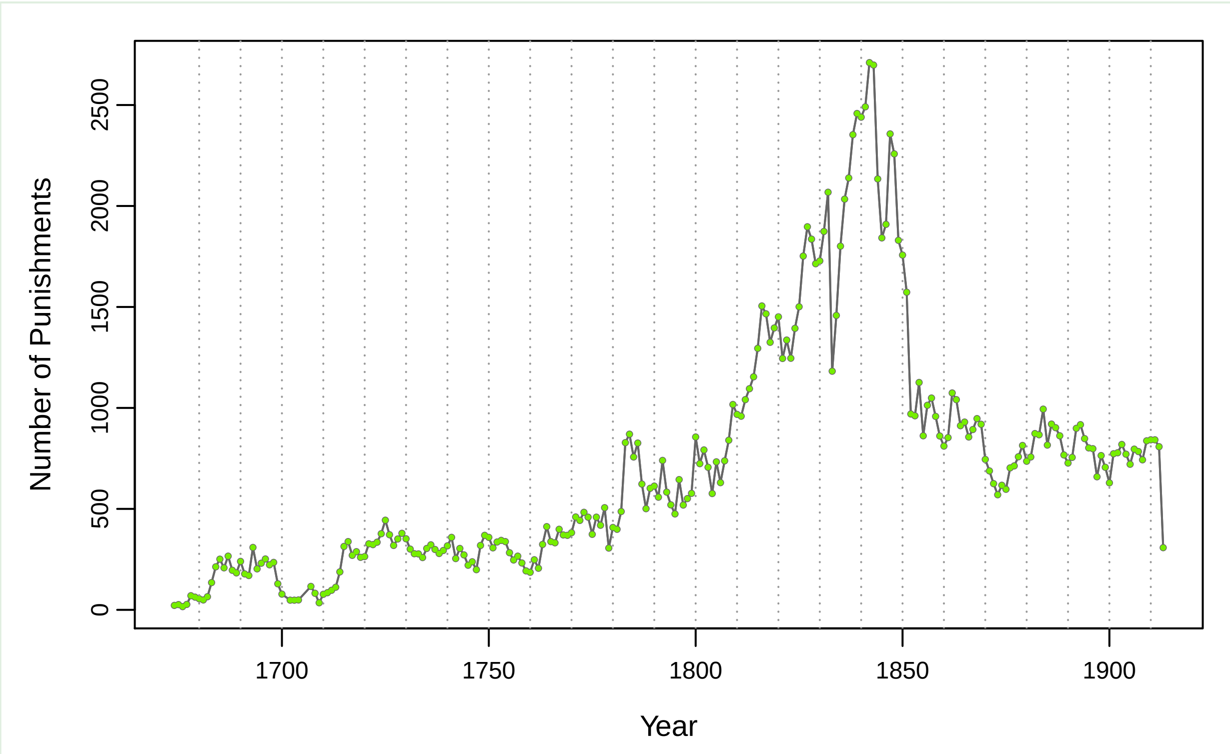


Figure 47: Number of Punishments at the Old Bailey, counting by punishments per year (Old Bailey Online 2018b).

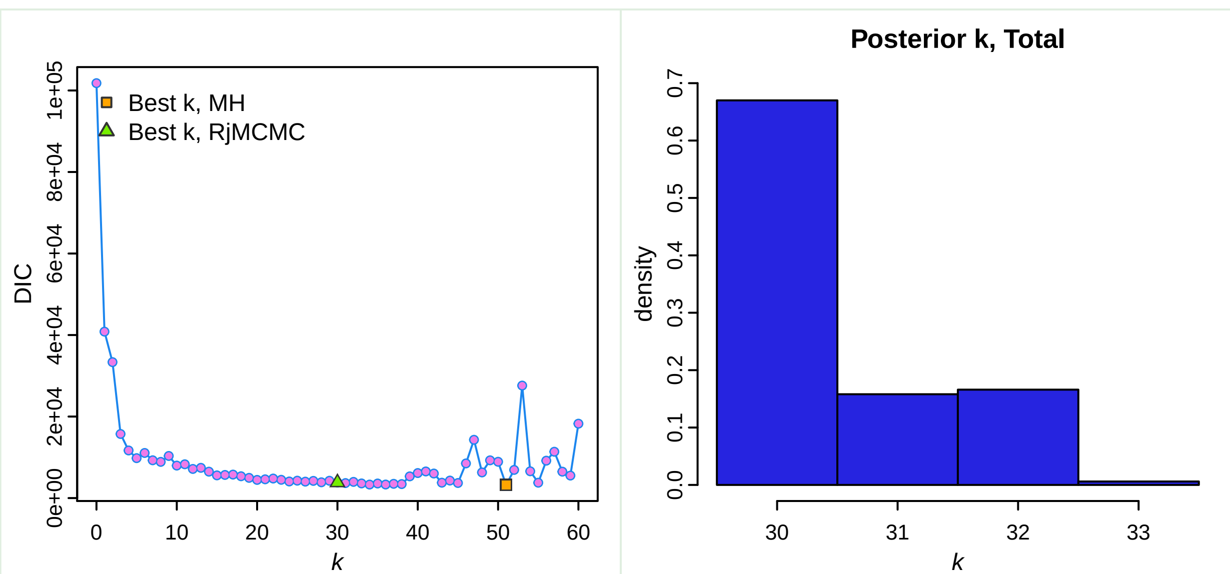


Figure 48: Model Estimates - DIC vs. Posterior k, Punishments by Offence, All Punishments (Old Bailey Online 2018b).

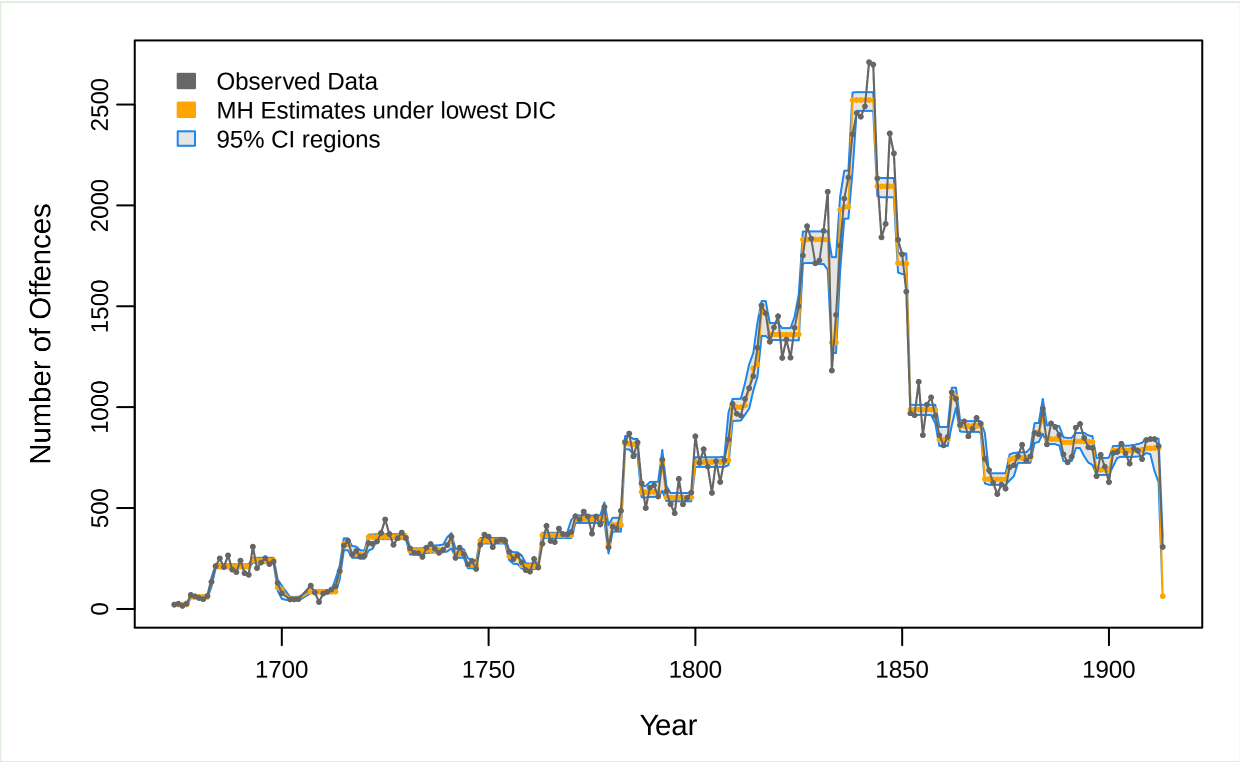


Figure 49: MH Estimates per Year, Punishments by Offence, All Punishments (Old Bailey Online 2018b).

k	DIC
0	101793.4
1	40830.9
2	33346.7
3	15725.3
4	11682.4
5	9797.6
6	11067.3
7	9272.2
8	8879.8
9	10334.1
10	7968.9
11	8287.8
12	7147.3
13	7418.6
14	6479.4
15	5567.3
16	5672.2
17	5762.3
18	5353.1
19	4960.9
20	4459.7
21	4601.8
22	4791.2
23	4480.9
24	4070.1
25	4267.8
26	4032.2
27	4218.0
28	3871.1
29	4246.7
30	3811.4
31	3678.3
32	3972.8
33	3592.8
34	3314.1
35	3571.5
36	3323.5
37	3496.2
38	3431.4
39	5325.3
40	6136.2
41	6533.4
42	6020.7
43	3771.1
44	4290.4
45	3695.5
46	8500.4
47	14299.4
48	6292.1
49	9262.1
50	8903.5
51	3221.8
52	6908.6
53	27595.3
54	6571.3
55	3770.1
56	9172.8
57	11368.1
58	6475.8
59	5519.6
60	18250.9

Table 66: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	22.5	(18.1, 27.5)
1	60.3	(53.4, 67.3)
2	133	(111.6, 155.2)
3	211.3	(202.2, 221.8)
4	241.6	(230.5, 254.4)
5	106.7	(91.1, 145.7)
6	49.9	(41.5, 59.8)
7	85.8	(78.9, 93.1)
8	184.7	(146.8, 215.1)
9	321.3	(291.4, 350.6)
10	269.7	(250.1, 290.6)
11	357.5	(346.2, 370.7)
12	289.7	(270.5, 300.9)
13	336.4	(301, 376.5)
14	275	(253.7, 296.6)
15	218.1	(199.3, 238.5)
16	338	(325.1, 352)
17	261.9	(242.2, 285)
18	212.1	(198.4, 226.1)
19	365.2	(351.4, 378.8)
20	446.5	(427.6, 464)
21	321.7	(275.2, 528.9)
22	417.5	(384.3, 452.8)
23	819.6	(791.3, 856.7)
24	580.1	(555, 807.4)
25	720.5	(583.8, 788.1)
26	552.7	(533.8, 574.4)
27	729.1	(704.9, 752)
28	998.2	(805.8, 1036.9)
29	1190.1	(971.6, 1258.8)
30	1467.2	(1094.5, 1524.7)
31	1360	(1331.5, 1391.6)
32	1830.7	(1431, 1868.1)
33	1320.9	(1268.2, 1743)
34	1993.3	(1934.7, 2172.5)
35	2522.5	(2468.9, 2561.5)
36	2094.5	(2040, 2136.7)
37	1713.5	(1659.9, 1762.1)
38	988.3	(962, 1012.9)
39	839.8	(808.4, 902.5)
40	1050.8	(996.2, 1097.5)
41	907.4	(879, 930.5)
42	643.1	(616.3, 672.5)
43	749.1	(725.3, 776.3)
44	874.7	(823.7, 920.6)
45	946.4	(733.8, 1041.3)
46	830.5	(797.4, 873.8)
47	690.4	(664.8, 748.3)
48	787	(754.9, 810.1)
49	318.4	(279.5, 845.2)
50	24.5	(18.8, 333.2)
51	57.9	(19.7, 66.6)

Table 67: MH Posterior Estimates for h, conditioned on k = 51

s	Posterior Estimate	95% CI	Year
1	4.5	(4, 5)	1679
2	9.4	(9, 10)	1684
3	10.6	(10.1, 11)	1685
4	19.5	(19, 20)	1694
5	25.4	(25, 26)	1700
6	27.6	(26.1, 28.9)	1702
7	33	(31.1, 34)	1707
8	40.4	(39.3, 41)	1715
9	41.6	(41, 42)	1716
10	43.7	(43, 45.8)	1718
11	47.6	(47, 49.3)	1722
12	57.4	(57.1, 58)	1732
13	66.6	(61.2, 67.8)	1741
14	68.5	(68, 68.9)	1743
15	71.5	(71, 72.8)	1746
16	74.5	(74.1, 75)	1749
17	81.5	(81, 82)	1756
18	84.7	(82.2, 85.9)	1759
19	89.5	(89, 90)	1764
20	97.4	(96.9, 98)	1772
21	105.1	(102.7, 105.9)	1780
22	106.1	(105, 106.9)	1781
23	109.5	(109, 110)	1784
24	113.5	(111.1, 114)	1788
25	118.2	(113.1, 118.9)	1793
26	119.6	(119, 120.7)	1794
27	126.6	(126, 127)	1801
28	135.4	(134.2, 136)	1810
29	139.8	(135.2, 140.9)	1814
30	142.3	(138.5, 142.9)	1817
31	144.6	(141.3, 147.2)	1819
32	152.4	(150.7, 153)	1827
33	159.4	(152.2, 160)	1834
34	161.6	(161, 162.7)	1836
35	164.6	(164, 165.6)	1839
36	170.5	(170, 171)	1845
37	175.5	(175, 176)	1850
38	178.6	(178.1, 179)	1853
39	185.5	(185, 186)	1860
40	188.4	(188, 189)	1863
41	190.5	(190, 191)	1865
42	196.4	(196, 197)	1871
43	202.7	(202, 204.4)	1877
44	208.5	(208, 209)	1883
45	210.6	(210, 215.8)	1885
46	211.9	(211.1, 218.8)	1886
47	223.4	(221.3, 223.9)	1898
48	227.5	(227, 229)	1902
49	239.1	(233, 239.5)	1913
50	239.4	(239, 240.1)	1913
51	240	(239.4, 240.8)	1913

Table 68: MH Posterior Estimates for s, conditioned on k = 51

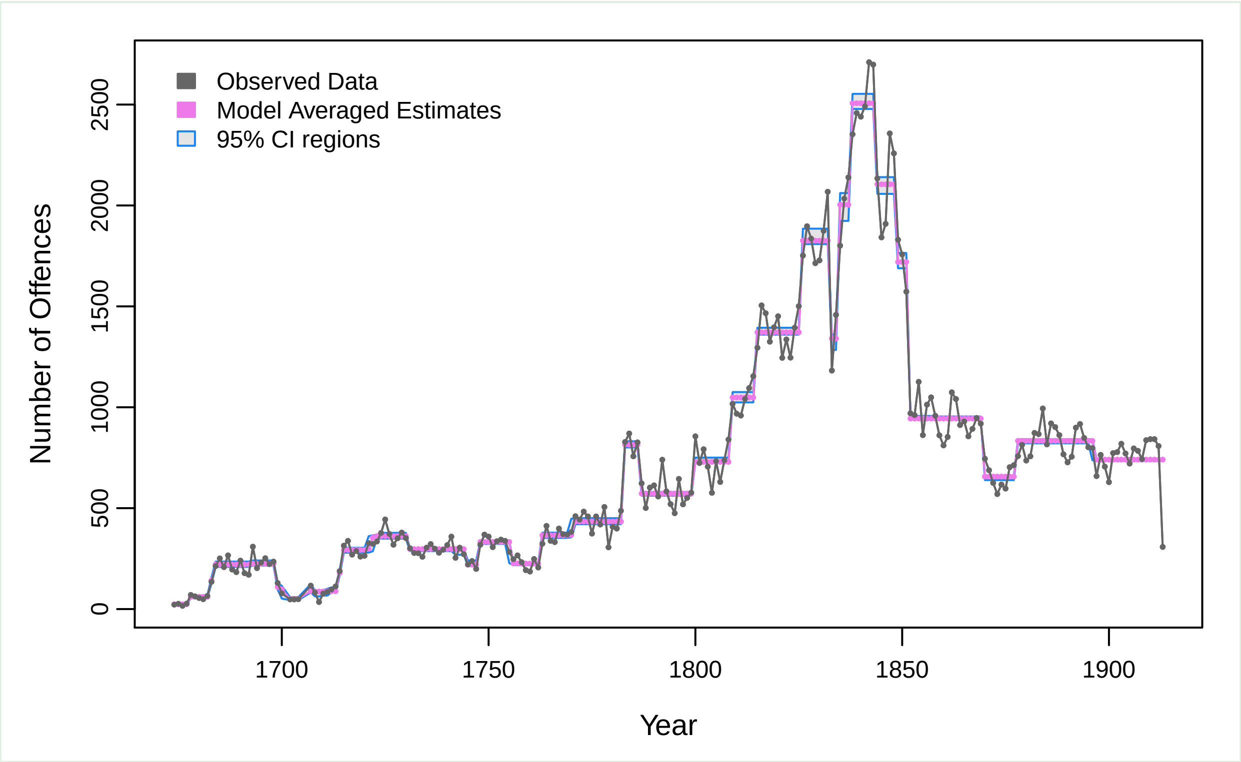


Figure 50: Model Averaged Estimates per Year, Punishments by Offence, All Punishments (Old Bailey Online 2018b).

k	Proportion
30	0.670
31	0.158
32	0.166
33	0.006

Table 69: Posterior estimate for k

h	Posterior Estimate	95% CI
0	23.7	(19.7, 27.3)
1	60.6	(56.8, 67.5)
2	142	(131.7, 153.9)
3	221.2	(215.8, 235.3)
4	108.5	(96.5, 128.4)
5	49.9	(44.7, 58.8)
6	87.8	(82.6, 92.7)
7	178.1	(165.8, 195.4)
8	292.4	(285.6, 303.7)
9	361.3	(349.2, 367.5)
10	297.3	(287.2, 297.3)
11	219.6	(213.4, 235.7)
12	332.8	(322.9, 336.8)
13	226.4	(220.2, 231.1)
14	363.8	(346.7, 378.6)
15	433.7	(420.7, 450.4)
16	816.6	(801.1, 840.1)
17	573.3	(562.1, 579.7)
18	729.4	(726.4, 750.5)
19	1048	(1033.5, 1075.4)
20	1370.9	(1360, 1394)
21	1825.7	(1808.7, 1862.8)
22	1340.9	(1267.5, 1360.2)
23	1998.3	(1923.4, 2061.5)
24	2506.7	(2478.1, 2514.2)
25	2104.9	(2057.3, 2140.4)
26	1731.4	(1713.8, 1764.5)
27	944.2	(944.2, 951.5)
28	656.5	(639.3, 660.9)
29	833.7	(833.1, 841.2)
30	740.6	(734.5, 743)

Table 70: Posterior Estimates for h, conditioned on k = 30

s	Posterior Estimate	95% CI	Year
1	4.3	(4, 4.9)	1679
2	9.3	(9, 9.8)	1684
3	10.5	(10.4, 10.9)	1685
4	25.4	(25.1, 25.9)	1700
5	27.3	(26.5, 28.6)	1702
6	32.2	(31.4, 33.9)	1707
7	40.3	(40, 40.9)	1715
8	41.6	(41.4, 42)	1716
9	48.7	(47, 50)	1723
10	57.4	(57.2, 57.7)	1732
11	71.4	(71, 71.8)	1746
12	74.5	(74.1, 74.9)	1749
13	82.7	(81.8, 82.9)	1757
14	89.3	(89, 89.8)	1764
15	97.4	(96, 98)	1772
16	109.7	(109.1, 110)	1784
17	113.4	(113.1, 114)	1788
18	126.5	(126.1, 126.9)	1801
19	135.9	(135.1, 136)	1810
20	141.5	(141.1, 141.9)	1816
21	152.5	(152, 152.7)	1827
22	159.3	(159.2, 159.9)	1834
23	161.5	(161.1, 161.9)	1836
24	164.4	(164.1, 164.8)	1839
25	170.7	(170.2, 171)	1845
26	175.7	(175, 175.8)	1850
27	178.7	(178.2, 178.9)	1853
28	196.5	(196.1, 196.9)	1871
29	204.4	(204, 205)	1879
30	223.2	(222.2, 224)	1898

Table 71: Posterior Estimates for s, conditioned on k = 30

Figures and Tables - Punishments by Offence - Death - (Old Bailey Online 2018b)

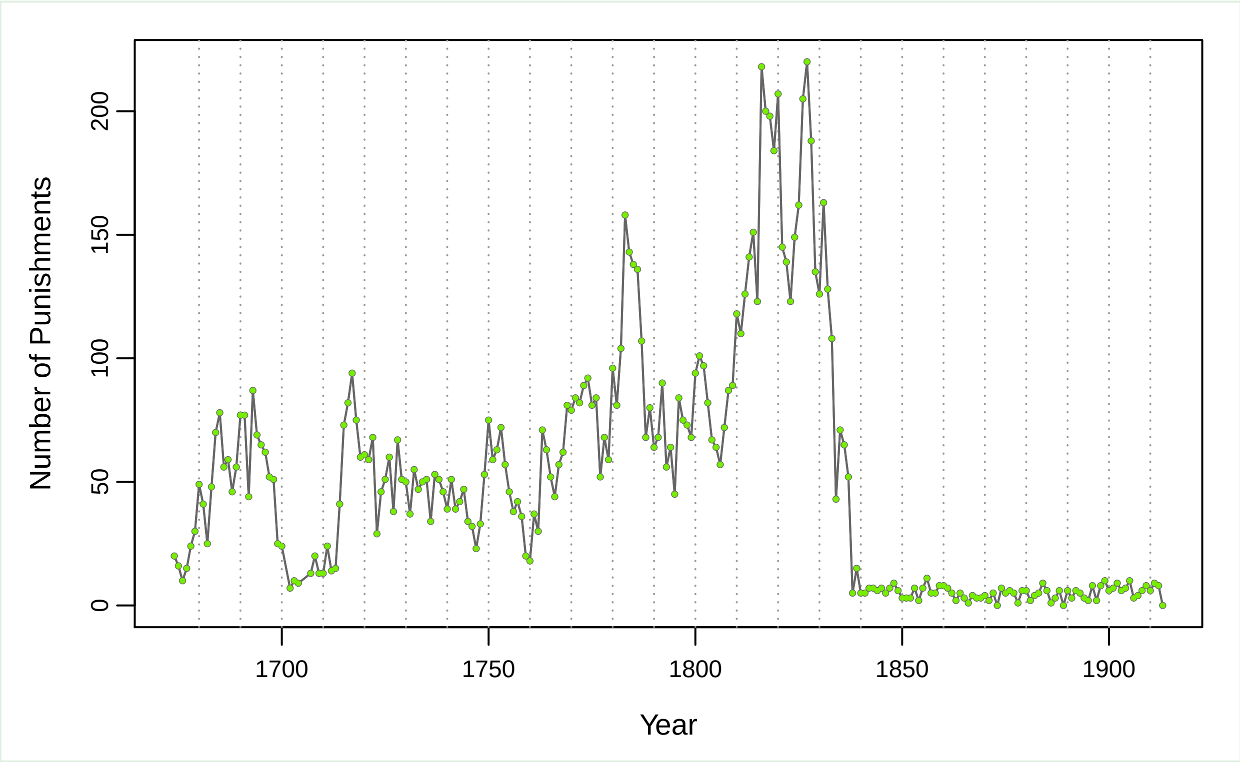
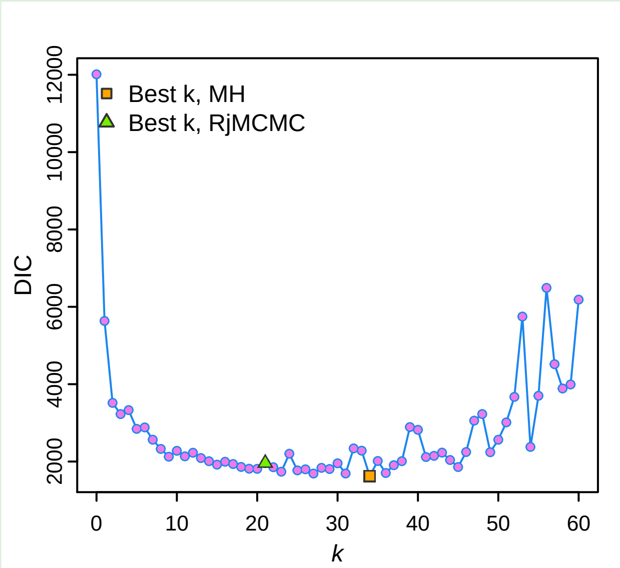


Figure 51: Number of Punishments of Death at the Old Bailey, counting by punishments per year (Old Bailey Online 2018b).



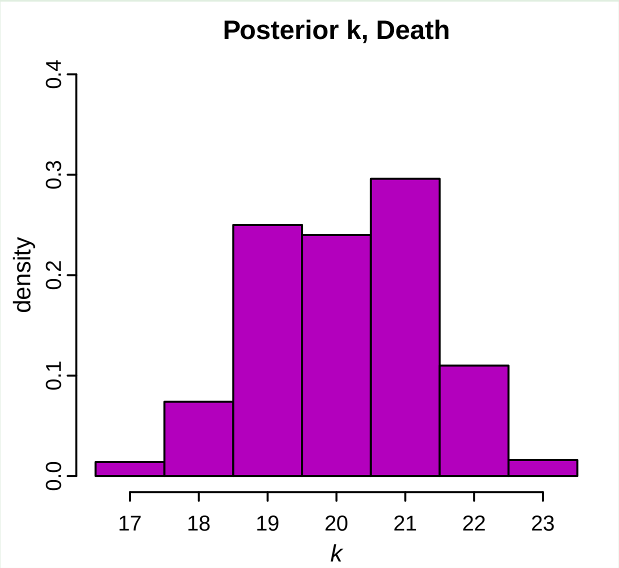


Figure 52: Model Estimates - DIC vs. Posterior k, Punishments by Offence, Death (Old Bailey Online 2018b).

Figures and Tables - Punishments by Offence - Death - MH - (Old Bailey Online 2018b)

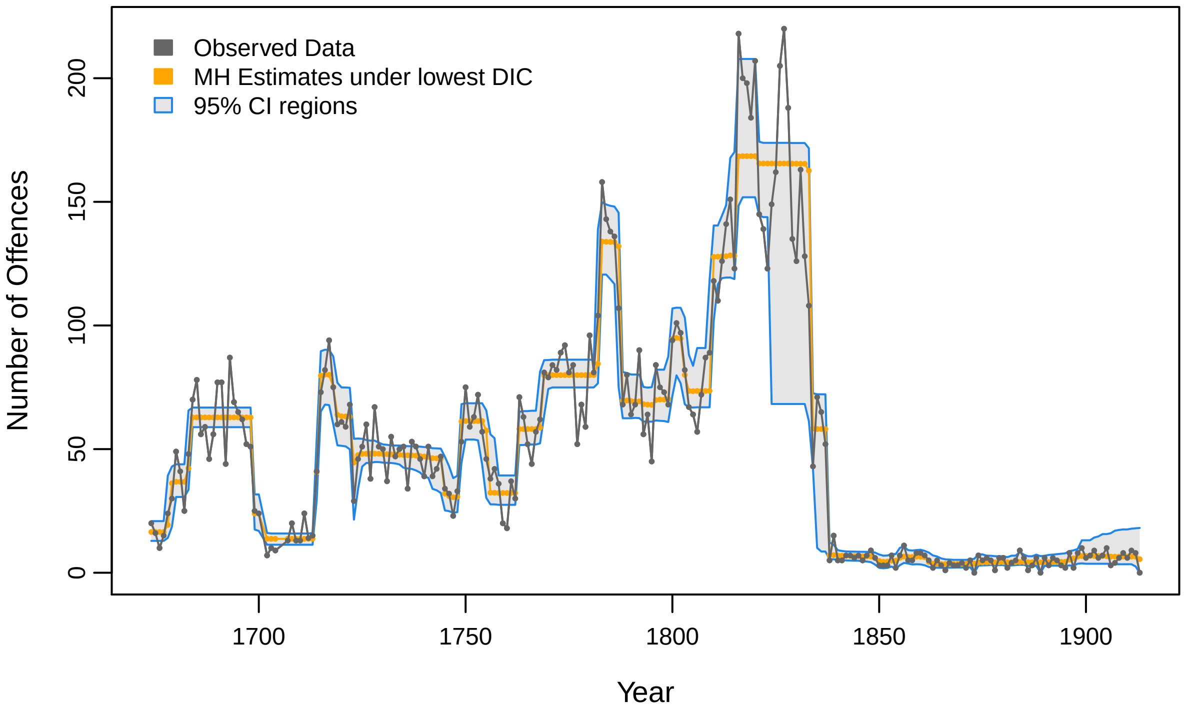


Figure 53: MH Estimates per Year, Punishments by Offence, Death (Old Bailey Online 2018b).

k	DIC
0	12012.0
1	5634.6
2	3516.6
3	3226.4
4	3330.0
5	2844.2
6	2882.5
7	2564.3
8	2326.1
9	2124.9
10	2277.5
11	2134.5
12	2227.3
13	2090.1
14	2008.3
15	1919.3
16	1994.3
17	1935.4
18	1860.5
19	1814.0
20	1810.6
21	1960.1
22	1853.0
23	1737.0
24	2201.4
25	1771.1
26	1798.7
27	1689.4
28	1836.8
29	1806.6
30	1955.0
31	1690.4
32	2339.3
33	2280.7
34	1623.8
35	2013.9
36	1704.0
37	1905.4
38	2009.2
39	2891.1
40	2819.7
41	2117.0
42	2147.0
43	2230.7
44	2039.2
45	1857.3
46	2245.7
47	3057.5
48	3227.1
49	2240.4
50	2563.5
51	3012.0
52	3671.4
53	5748.7
54	2377.1
55	3700.4
56	6490.2
57	4516.7
58	3888.0
59	3992.6
60	6186.1

Table 72: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	16.5	(12.9, 20.9)
1	36.8	(30.6, 43.9)
2	62.8	(58.9, 66.8)
3	23.9	(17.5, 31.7)
4	13.7	(11.3, 15.9)
5	40.6	(29.3, 54.3)
6	80.2	(68, 90.2)
7	61.8	(26.7, 71.9)
8	46.5	(22.5, 51.4)
9	45.7	(26, 51)
10	30.9	(24.5, 56.7)
11	61.4	(53.9, 68.5)
12	32.3	(27.5, 39.3)
13	58.1	(51.9, 65.5)
14	79.9	(74.9, 86.2)
15	133.9	(120.6, 149.9)
16	70.3	(62.5, 118.7)
17	70.1	(60.8, 86.7)
18	94.9	(79.3, 107.2)
19	73.5	(66.9, 90.9)
20	127.9	(119.1, 163.2)
21	168.5	(151.9, 207.8)
22	61.5	(45.3, 158.3)
23	7.5	(5.4, 62.3)
24	5.3	(2, 8.2)
25	5.9	(1.5, 11.2)
26	6	(2, 9.1)
27	3.6	(0, 5.9)
28	4.4	(2.8, 8.9)
29	5.8	(0, 8.9)
30	5.2	(0, 14.3)
31	14.4	(2.5, 33.1)
32	33	(0, 60.1)
33	60.1	(0, 66.5)
34	24.4	(13.4, 65.2)

Table 73: MH Posterior Estimates for h, conditioned on k = 34

s	Posterior Estimate	95% CI	Year
1	5.1	(4.2, 6.5)	1680
2	10.1	(9.2, 11)	1685
3	25.4	(25, 25.9)	1700
4	28.2	(27, 29)	1703
5	40.4	(40, 41)	1715
6	41.5	(41.1, 42)	1716
7	45.3	(44.1, 49.7)	1720
8	49.6	(48.6, 56.2)	1724
9	61.2	(50.2, 71.9)	1736
10	71.7	(68, 75.6)	1746
11	75.6	(75, 76.6)	1750
12	82.2	(81, 84.3)	1757
13	89.5	(89.1, 90)	1764
14	95.5	(94.4, 96.6)	1770
15	109.1	(108, 109.9)	1784
16	114.4	(112.8, 115)	1789
17	121.2	(114.2, 125.3)	1796
18	126.5	(126, 127.9)	1801
19	130	(128.2, 132.1)	1804
20	136.5	(136.1, 137.4)	1811
21	142.4	(140.4, 143)	1817
22	160.1	(147.1, 161)	1835
23	164.5	(160.2, 165)	1839
24	173.5	(164.1, 178.7)	1848
25	180.3	(174.5, 184.7)	1855
26	183.8	(178.9, 190)	1858
27	189.1	(185.2, 199.2)	1864
28	201.8	(191.6, 212)	1876
29	218.6	(202.7, 227)	1893
30	225.7	(208, 239.6)	1900
31	240	(213, 240.4)	1913
32	240.2	(222.2, 240.6)	1913
33	240.4	(234.8, 240.8)	1913
34	240.7	(240.1, 241)	1913

Table 74: MH Posterior Estimates for s, conditioned on k = 34

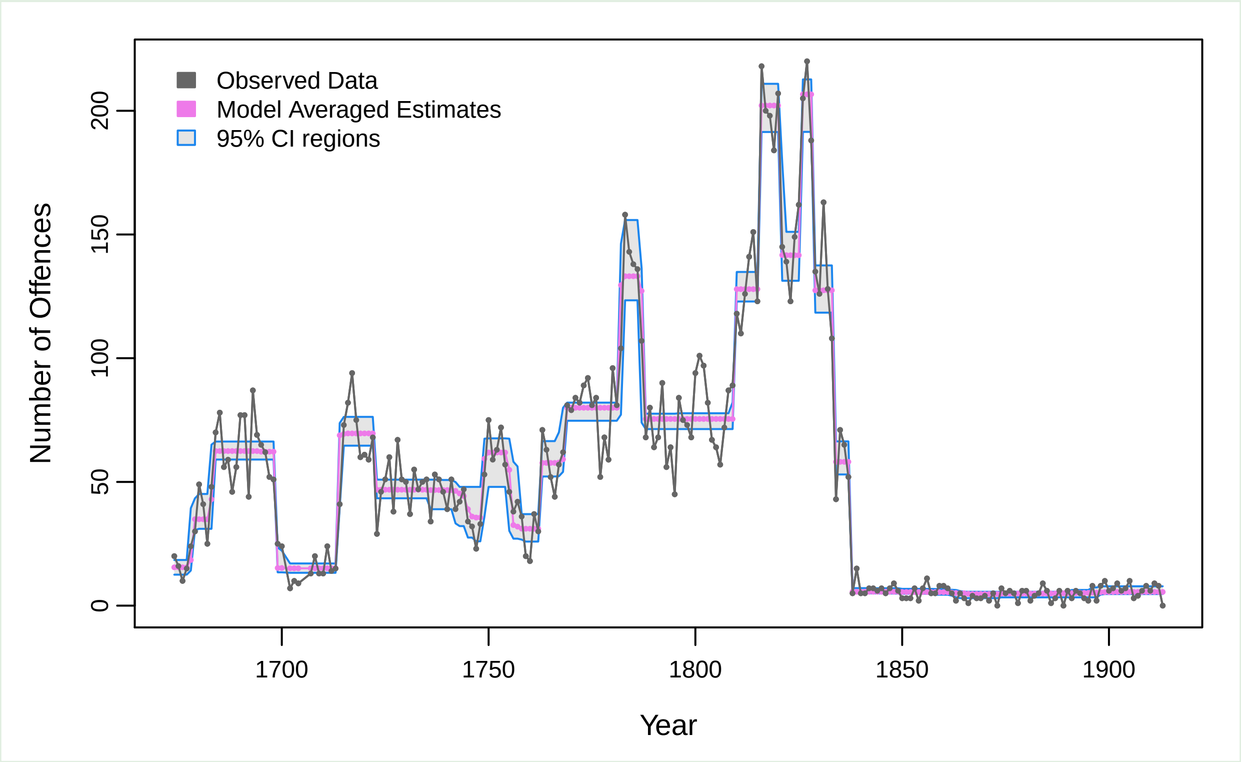


Figure 54: Model Averaged Estimates per Year, Punishments by Offence, Death (Old Bailey Online 2018b).

k	Proportion
17	0.014
18	0.074
19	0.250
20	0.240
21	0.296
22	0.110
23	0.016

Table 75: Posterior estimate for k

h	Posterior Estimate	95% CI
0	14.5	(12.5, 18.2)
1	36.8	(31.1, 47.1)
2	62.8	(59, 66.3)
3	15.8	(13.5, 39.8)
4	68.9	(13.6, 73.8)
5	47.2	(40.4, 75.2)
6	35.5	(28.1, 64.7)
7	61.8	(27, 67.5)
8	32.6	(26.7, 63.6)
9	58.6	(30.9, 66.7)
10	78.5	(55.1, 83)
11	131.5	(72.4, 135.9)
12	75.4	(70.3, 135.8)
13	126.6	(74.2, 136.5)
14	194	(75.8, 211.4)
15	141.6	(131.3, 203.8)
16	201.3	(135.5, 212.7)
17	130.3	(122, 209.9)
18	58.6	(53.6, 203.5)
19	5.9	(5.1, 123.6)
20	4.2	(3.2, 60.2)
21	6.5	(5, 7.9)

Table 76: Posterior Estimates for h, conditioned on k = 21

s	Posterior Estimate	95% CI	Year
1	5.1	(4.1, 5.9)	1680
2	10.5	(9, 11)	1685
3	25.6	(24.5, 25.8)	1700
4	40.6	(26.9, 40.9)	1715
5	49.4	(40.2, 50)	1724
6	69.4	(41.5, 73.4)	1744
7	75.3	(50, 77)	1750
8	82.3	(75.2, 84.5)	1757
9	89.2	(81.1, 90)	1764
10	95.5	(89, 95.9)	1770
11	108.2	(94.4, 109.5)	1783
12	114.8	(108.1, 114.9)	1789
13	136.1	(108.6, 136.9)	1811
14	142.4	(114.7, 142.8)	1817
15	147.5	(136.8, 147.9)	1822
16	152.5	(142.7, 152.9)	1827
17	155.4	(147.7, 155.9)	1830
18	160.6	(152.8, 161)	1835
19	164.3	(155.8, 164.9)	1839
20	189	(160.6, 194.8)	1863
21	222.7	(164.6, 225.9)	1897

Table 77: Posterior Estimates for s, conditioned on k = 21

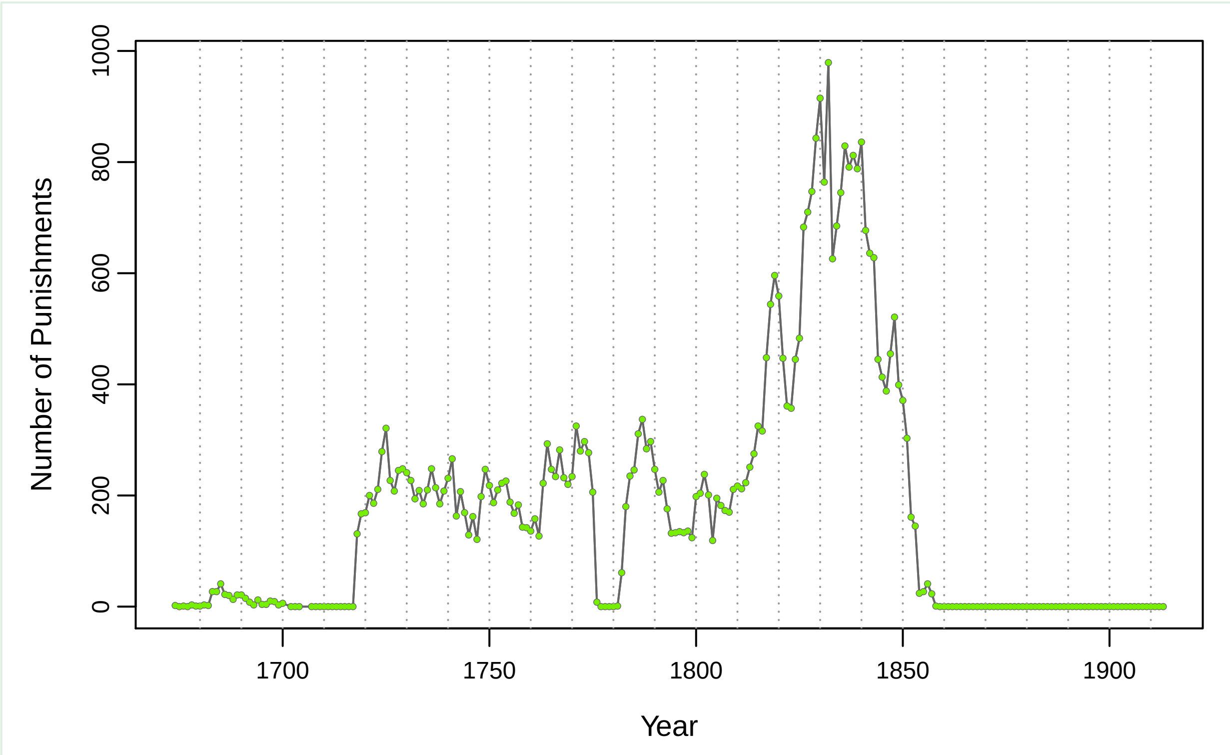


Figure 55: Number of Punishments of Transportation at the Old Bailey, counting by punishments per year (Old Bailey Online 2018b).

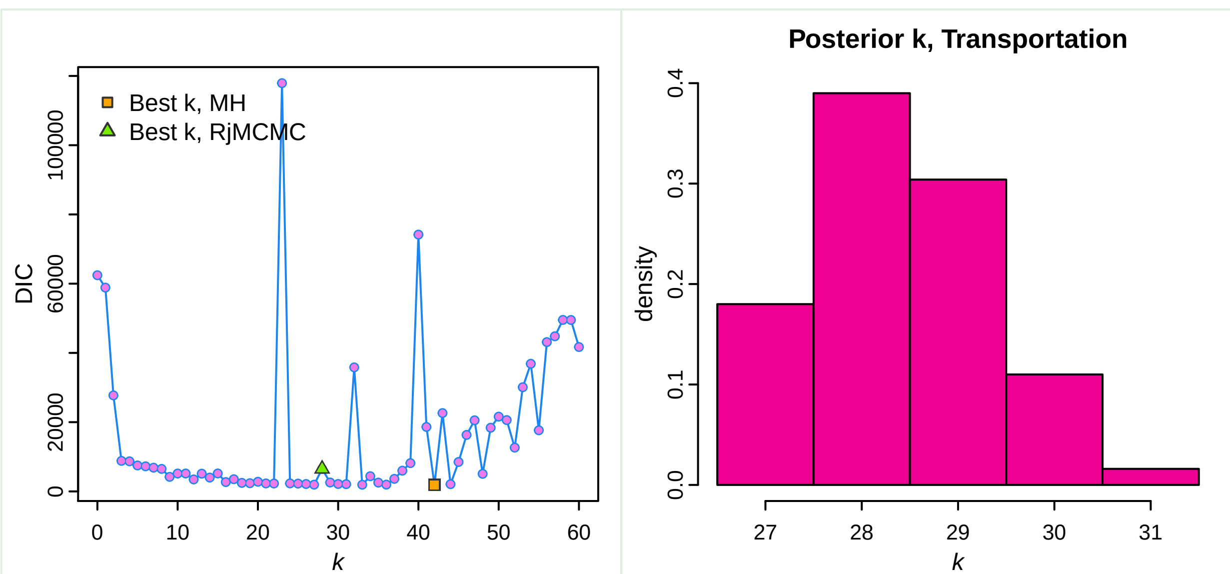


Figure 56: Model Estimates - DIC vs. Posterior k, Punishments by Offence, Transportation (Old Bailey Online 2018b).

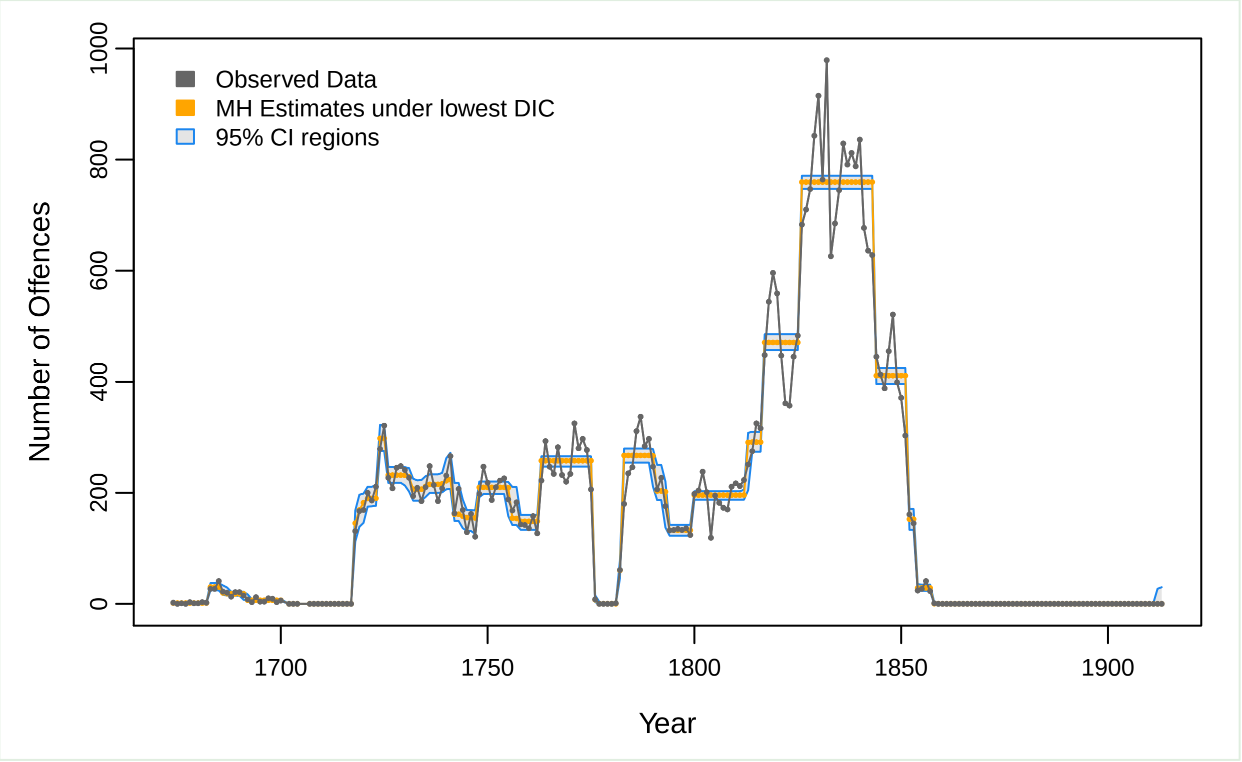


Figure 57: MH Estimates per Year, Punishments by Offence, Transportation (Old Bailey Online 2018b).

k	DIC
0	62438.4
1	58859.1
2	27728.6
3	8821.0
4	8690.9
5	7493.5
6	7248.3
7	6851.3
8	6498.8
9	4201.3
10	5160.6
11	5171.0
12	3458.3
13	5123.1
14	3991.9
15	5183.2
16	2698.2
17	3520.2
18	2457.9
19	2366.4
20	2787.8
21	2302.1
22	2277.4
23	117916.1
24	2311.4
25	2256.6
26	2137.8
27	1944.4
28	6486.8
29	2569.5
30	2144.2
31	2089.6
32	35825.2
33	1928.2
34	4383.8
35	2543.5
36	1959.3
37	3621.3
38	5986.6
39	8163.0
40	74171.8
41	18618.1
42	1865.9
43	22616.8
44	2090.6
45	8498.7
46	16317.3
47	20533.9
48	5071.9
49	18384.9
50	21592.0
51	20596.2
52	12642.6
53	30089.9
54	36866.2
55	17655.5
56	43124.1
57	44803.1
58	49531.0
59	49513.4
60	41699.2

Table 78: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	1.4	(0.8, 2.2)
1	30.3	(24.5, 37.3)
2	18.1	(14.6, 22)
3	6.5	(4.9, 8.3)
4	0	(0, 0)
5	145.2	(112.1, 168)
6	189.8	(175.6, 210.9)
7	298.1	(275.3, 322.5)
8	231.9	(217.8, 246.4)
9	206.4	(185.9, 222.8)
10	223	(166.4, 272.1)
11	154.8	(125.9, 168.3)
12	209.8	(197.6, 220.3)
13	148.6	(133.3, 160.1)
14	257.8	(247.2, 265.7)
15	7.4	(2.5, 14.5)
16	0.1	(0, 0.7)
17	60.2	(45.9, 78.1)
18	267.3	(254.4, 279.7)
19	202.8	(182.7, 222.1)
20	132.5	(122.8, 142.1)
21	195.8	(187.7, 202.7)
22	291.2	(274.1, 309.8)
23	470.7	(456.9, 485.4)
24	759.5	(747.4, 771)
25	410.8	(395.9, 424.7)
26	152.4	(133.3, 170.7)
27	28.6	(23.3, 34.6)
28	0.3	(0, 2.5)
29	0	(0, 0)
30	0	(0, 0)
31	0	(0, 0)
32	0	(0, 0)
33	0	(0, 0)
34	0	(0, 0)
35	0	(0, 0)
36	0	(0, 0)
37	0	(0, 0)
38	0	(0, 0)
39	0	(0, 0)
40	0	(0, 1.1)
41	0	(0, 29.8)
42	1.5	(0, 35.1)

Table 79: MH Posterior Estimates for h, conditioned on k = 42

s	Posterior Estimate	95% CI	Year
1	9.5	(9, 10)	1684
2	12.8	(12.1, 14.6)	1687
3	18.6	(17.5, 19.2)	1693
4	28	(27.1, 29)	1702
5	44.4	(44, 45)	1719
6	46.1	(45.1, 47.9)	1721
7	50.4	(50, 51)	1725
8	52.6	(52, 53)	1727
9	58.2	(55.1, 59)	1733
10	64.8	(61, 68.9)	1739
11	68.8	(68, 73)	1743
12	74.5	(74.1, 75)	1749
13	82.7	(81.9, 85)	1757
14	89.5	(89, 90)	1764
15	102.4	(102, 103)	1777
16	103.6	(103, 104.8)	1778
17	108.4	(108, 108.9)	1783
18	109.6	(109.1, 110)	1784
19	117.5	(116.5, 119)	1792
20	120.5	(119.8, 121)	1795
21	126.5	(126, 127)	1801
22	139.5	(139, 140.3)	1814
23	143.6	(143, 143.9)	1818
24	152.5	(152, 153)	1827
25	170.5	(170, 171)	1845
26	178.4	(178, 178.9)	1853
27	180.6	(180, 181)	1855
28	184.4	(184, 185)	1859
29	186.5	(185.2, 190.4)	1861
30	190.2	(187, 194.5)	1865
31	194.8	(190.2, 201.1)	1869
32	200	(192.7, 208)	1874
33	204.3	(196.6, 211.3)	1879
34	208.3	(200.5, 214.7)	1883
35	210.9	(203.7, 218.9)	1885
36	214.2	(207.8, 223.3)	1889
37	220.1	(210, 227.8)	1895
38	224.6	(215.3, 229.9)	1899
39	228.2	(218.5, 237)	1903
40	233.3	(225.3, 239.3)	1908
41	237.3	(228.9, 240.3)	1912
42	240.2	(236.7, 240.8)	1913

Table 80: MH Posterior Estimates for s, conditioned on k = 42

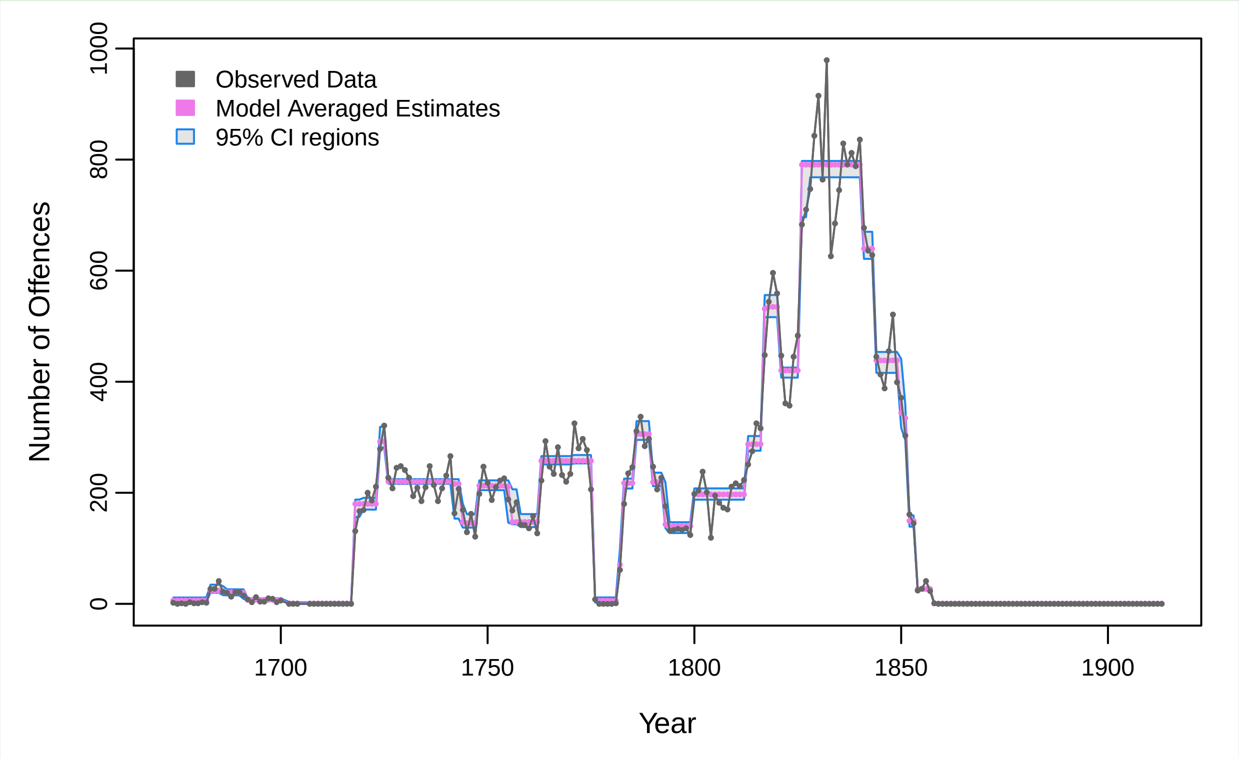


Figure 58: Model Averaged Estimates per Year, Punishments by Offence, Transportation (Old Bailey Online 2018b).

k	Proportion
27	0.180
28	0.390
29	0.304
30	0.110
31	0.016

Table 81: Posterior estimate for k

h	Posterior Estimate	95% CI
0	6	(3, 58.7)
1	22.2	(6.3, 34.7)
2	7.8	(5.8, 21.5)
3	1.6	(0.4, 9.3)
4	2.9	(0.5, 183.2)
5	187.8	(169.7, 315.4)
6	285.7	(216.1, 318.7)
7	216.1	(137.3, 223.2)
8	153.7	(137, 216.7)
9	205	(145.3, 222.4)
10	155.4	(138.2, 260)
11	256.4	(2.6, 261.2)
12	8	(1.7, 80)
13	70.5	(64, 222.3)
14	221.4	(207.7, 309.1)
15	303	(218.9, 329.1)
16	218.9	(139.6, 240.8)
17	140.1	(127.5, 201.2)
18	197	(187.6, 296.8)
19	292.1	(276.1, 534.8)
20	531.4	(410.3, 556.2)
21	421.9	(407.4, 791)
22	786.4	(639.5, 791)
23	630.9	(433.2, 670.1)
24	438	(301.3, 453.6)
25	335.8	(140.9, 358.8)
26	148.9	(26.1, 159.3)
27	26.8	(0.5, 29.4)
28	0.6	(0.4, 1.1)

Table 82: Posterior Estimates for h, conditioned on k = 28

s	Posterior Estimate	95% CI	Year
1	9.1	(0.6, 9.9)	1684
2	18	(9.1, 19)	1692
3	27.4	(17.6, 29)	1702
4	37.1	(27.6, 44.9)	1712
5	45	(44.1, 50.9)	1719
6	51	(50, 52.9)	1725
7	52.9	(52.3, 70.9)	1727
8	70.9	(68.1, 75)	1745
9	74.7	(74, 82.9)	1749
10	84.7	(82, 89.7)	1759
11	89.7	(89.3, 102.8)	1764
12	102.8	(102.1, 108.5)	1777
13	108.3	(108, 109.8)	1783
14	109.8	(109.1, 112.5)	1784
15	112.4	(112, 116.3)	1787
16	116.4	(116, 119.6)	1791
17	119.8	(119, 127)	1794
18	126.6	(126.1, 140)	1801
19	139.4	(139, 143.9)	1814
20	143.3	(143, 147.4)	1818
21	147.4	(147, 153)	1822
22	152.9	(152.1, 167.6)	1827
23	167.6	(167.1, 170.6)	1842
24	170.7	(170.1, 177.4)	1845
25	176.9	(176, 178.6)	1851
26	178.3	(178.1, 180.9)	1853
27	180.4	(180.1, 184.8)	1855
28	184.8	(184.2, 225.2)	1859

Table 83: Posterior Estimates for s, conditioned on k = 28

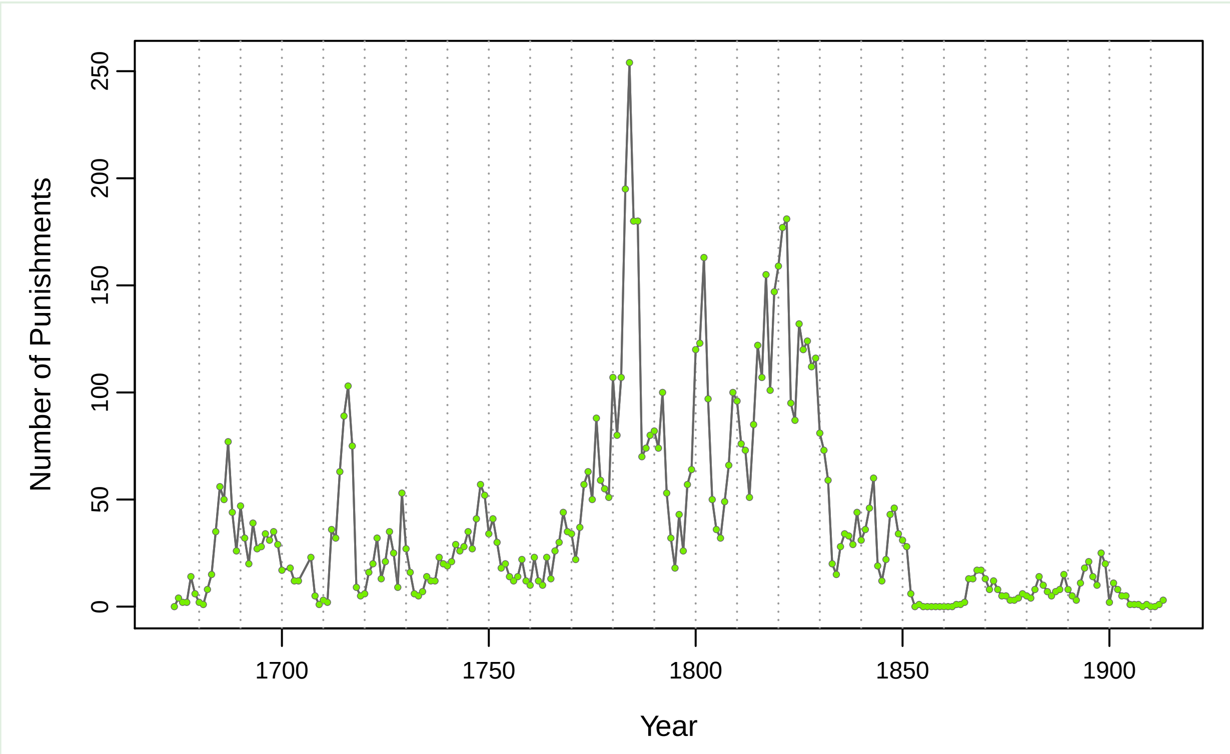


Figure 59: Number of Corporal Punishments at the Old Bailey, counting by punishments per year (Old Bailey Online 2018b).

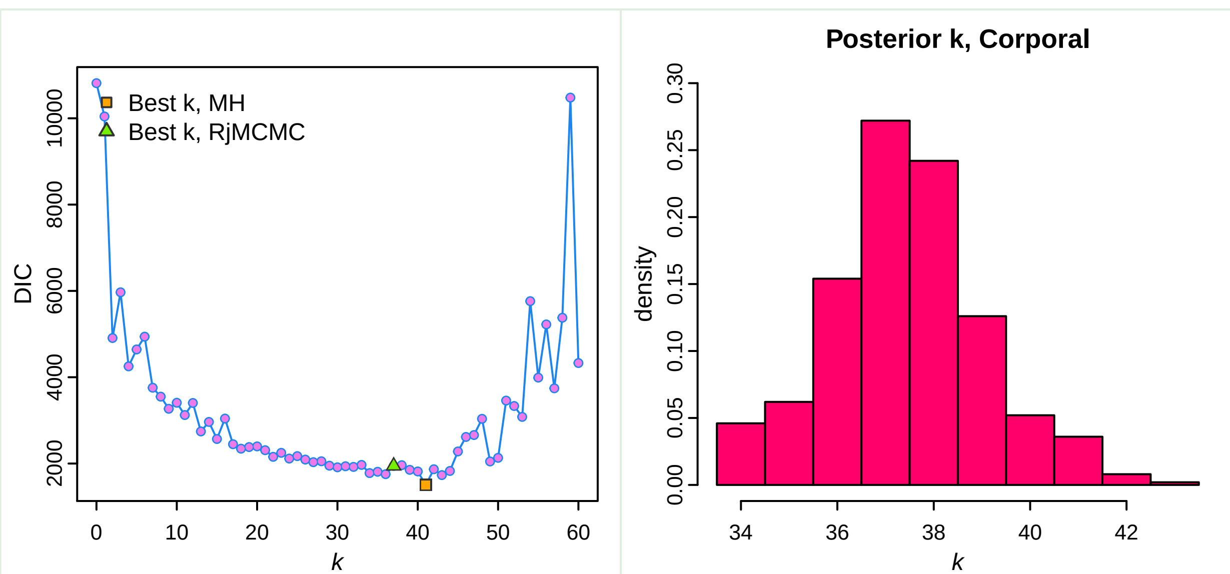


Figure 60: Model Estimates - DIC vs. Posterior k, Punishments by Offence, Corporal (Old Bailey Online 2018b).

Figures and Tables - Punishments by Offence - Corporal - MH - (Old Bailey Online 2018b)

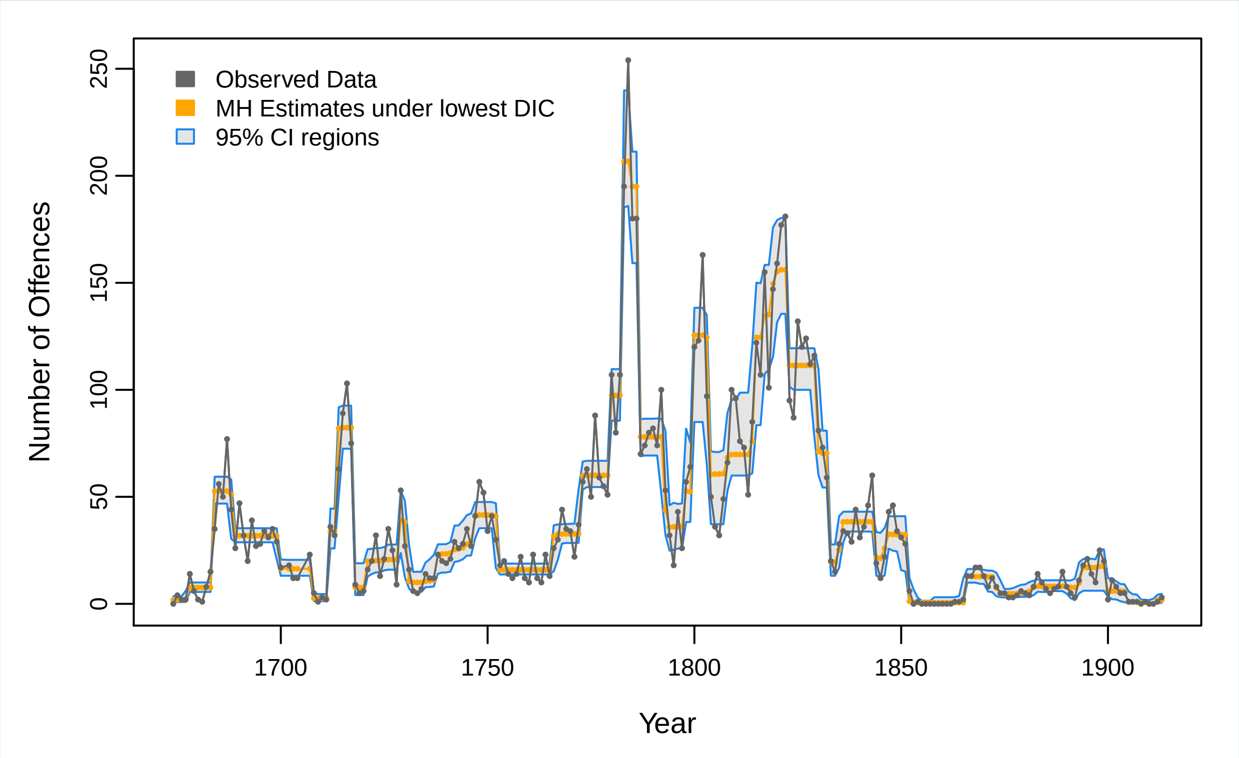


Figure 61: MH Estimates per Year, Punishments by Offence, Corporal (Old Bailey Online 2018b).

k	DIC
0	10814.0
1	10042.1
2	4908.5
3	5968.0
4	4252.2
5	4643.7
6	4940.2
7	3756.8
8	3550.0
9	3268.8
10	3408.5
11	3123.4
12	3403.4
13	2743.7
14	2963.9
15	2568.8
16	3041.9
17	2447.9
18	2344.3
19	2381.7
20	2397.7
21	2309.2
22	2155.5
23	2247.9
24	2115.3
25	2172.1
26	2092.0
27	2031.8
28	2052.6
29	1950.5
30	1912.7
31	1937.5
32	1922.0
33	1968.2
34	1779.0
35	1809.3
36	1753.9
37	1945.0
38	1962.1
39	1853.6
40	1816.4
41	1504.2
42	1867.6
43	1732.0
44	1828.0
45	2280.4
46	2616.0
47	2660.3
48	3036.3
49	2047.8
50	2132.3
51	3458.7
52	3333.1
53	3081.1
54	5762.5
55	3990.8
56	5223.3
57	3743.3
58	5378.5
59	10479.0
60	4328.9

Table 84: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	1.9	(0.8, 3.5)
1	7.6	(5.5, 10)
2	52.7	(46.9, 59.4)
3	31.8	(28.8, 35.3)
4	16.4	(13.1, 20.5)
5	2.6	(1.3, 4.6)
6	33.9	(25.9, 44.5)
7	82.3	(72.5, 92.6)
8	7.7	(4.1, 19)
9	23.6	(18.6, 51.2)
10	33.2	(5.4, 48.8)
11	11.7	(7.6, 24.9)
12	25.6	(18.2, 33.7)
13	41.5	(35.3, 47.6)
14	15.9	(13.7, 18.8)
15	32.7	(28.4, 37.4)
16	60.1	(54.5, 66.8)
17	97.6	(86, 110.3)
18	206.7	(185.2, 239.9)
19	80.9	(69, 198.7)
20	36.1	(24.9, 86.2)
21	51.8	(37.8, 72.8)
22	125.5	(85, 138.3)
23	60.8	(37.2, 71.8)
24	107.7	(73.5, 136.5)
25	156.1	(135.5, 180.3)
26	111.4	(100, 119.4)
27	70.3	(54.3, 80.9)
28	19.5	(13.1, 28.8)
29	38.4	(33.8, 43.1)
30	21.1	(12.4, 33.2)
31	31.7	(2.3, 40.9)
32	0.6	(0.2, 3.1)
33	12.8	(9.5, 16.2)
34	4.7	(3.1, 9.7)
35	8	(3.5, 11.8)
36	8.2	(2.2, 19.3)
37	17.5	(6.1, 25.5)
38	5.9	(0.7, 8.4)
39	0.7	(0, 2.6)
40	0	(0, 1.3)
41	1.8	(0.5, 4.7)

Table 85: MH Posterior Estimates for h, conditioned on k = 41

s	Posterior Estimate	95% CI	Year
1	4.5	(3.4, 5)	1679
2	10.5	(10, 11)	1685
3	15.4	(14.2, 16)	1690
4	26.5	(25.7, 27)	1701
5	34.4	(34, 35)	1709
6	38.4	(38, 39)	1713
7	40.5	(40, 41.2)	1715
8	44.6	(44, 45)	1719
9	47.7	(47.1, 56)	1722
10	55.9	(55, 58.5)	1730
11	57.9	(57, 65)	1732
12	64.8	(61.9, 70.1)	1739
13	73.3	(68.9, 74.3)	1748
14	79.4	(78.3, 80)	1754
15	92.7	(92, 94.8)	1767
16	99.5	(99, 100)	1774
17	106.5	(106, 107)	1781
18	109.4	(109, 109.9)	1784
19	113.1	(111.1, 113.9)	1788
20	119.2	(113.1, 120.9)	1794
21	124	(119, 124.9)	1798
22	126.5	(124.6, 127)	1801
23	130.4	(129.5, 131.6)	1805
24	140.1	(134.1, 141.9)	1815
25	144.5	(141.1, 146.5)	1819
26	149.5	(149, 150)	1824
27	156.6	(155.8, 157.5)	1831
28	159.4	(159, 160)	1834
29	162.1	(161, 163)	1837
30	170.5	(170, 171.1)	1845
31	173.9	(172.6, 178.9)	1848
32	178.9	(178, 180.6)	1853
33	192.5	(191.9, 193)	1867
34	199.9	(197.1, 201)	1874
35	207.4	(200.3, 209.7)	1882
36	217	(208.5, 220.9)	1891
37	220.5	(218.4, 225)	1895
38	226.5	(226, 229.2)	1901
39	231.7	(229.6, 234.5)	1906
40	236.3	(233.7, 237.9)	1911
41	238.5	(235.6, 239.8)	1913

Table 86: MH Posterior Estimates for s, conditioned on k = 41

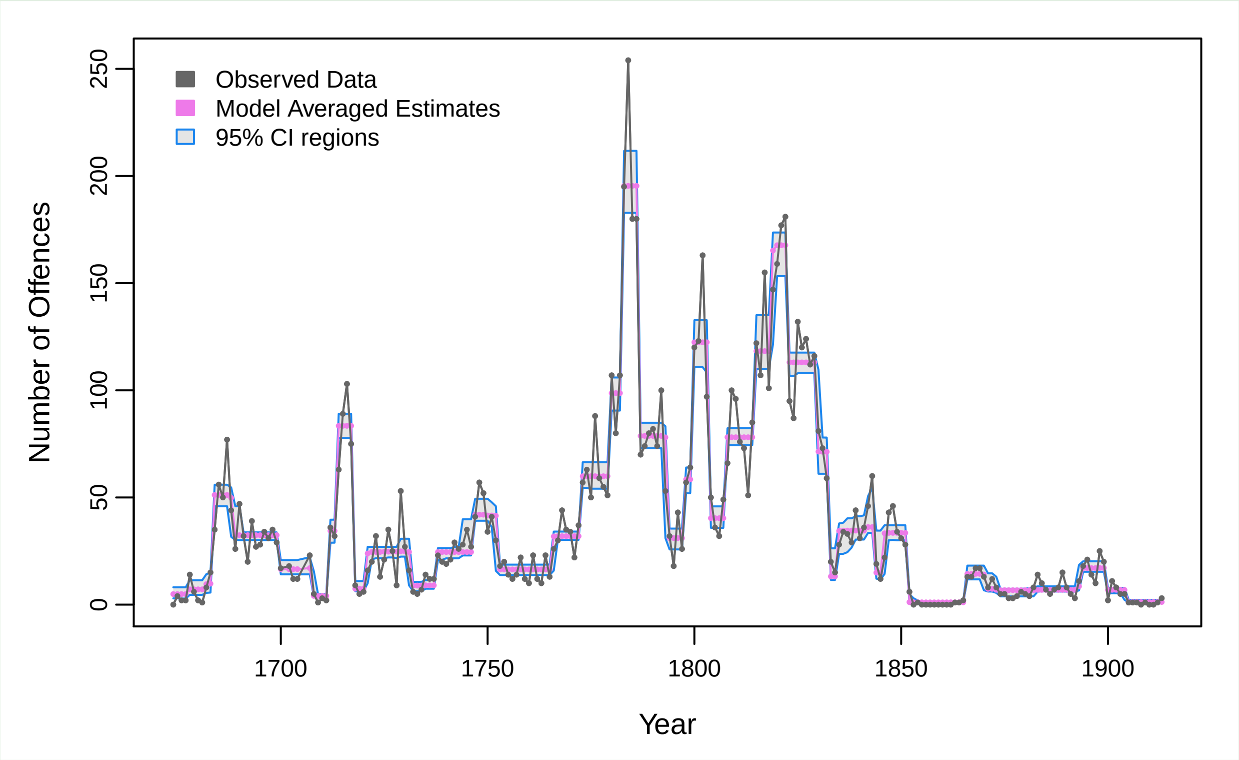


Figure 62: Model Averaged Estimates per Year, Punishments by Offence, Corporal (Old Bailey Online 2018b).

k	Proportion
34	0.046
35	0.062
36	0.154
37	0.272
38	0.242
39	0.126
40	0.052
41	0.036
42	0.008
43	0.002

Table 87: Posterior estimate for k

h	Posterior Estimate	95% CI
0	5	(2.8, 8.1)
1	10.9	(7.4, 56)
2	49.5	(31.6, 55.6)
3	32.4	(14.1, 34.3)
4	15.5	(3.6, 20.8)
5	4.1	(3.6, 32)
6	36.4	(30.6, 83.6)
7	81.2	(7.2, 89.1)
8	10	(6.3, 26.1)
9	24	(8.1, 26.7)
10	9.4	(7.2, 26.4)
11	24.8	(12.8, 45)
12	41.4	(15.7, 49.4)
13	18	(13.8, 49.4)
14	33.3	(16.1, 65.3)
15	59.9	(15.4, 98.7)
16	99.3	(32.8, 199.1)
17	192.3	(59.7, 211.6)
18	78.2	(25.8, 192.3)
19	33.8	(25.8, 200.7)
20	58.4	(30.2, 127.2)
21	120	(25.8, 131.4)
22	43.6	(37, 106.1)
23	82.2	(43.6, 122)
24	120.8	(39.9, 173.6)
25	155.2	(76.3, 172)
26	113	(68.1, 156.2)
27	69.7	(13.1, 168.9)
28	23.7	(11.5, 110.8)
29	36.2	(12.3, 77.4)
30	13.8	(4.9, 37.1)
31	33.3	(0.8, 36.7)
32	1.1	(0.8, 16.6)
33	13	(5.5, 17)
34	6.7	(6.3, 9.6)
35	17.1	(15.3, 19.3)
36	7.1	(5.4, 7.8)
37	1.3	(0.9, 2.2)

Table 88: Posterior Estimates for h, conditioned on k = 37

s	Posterior Estimate	95% CI	Year
1	8.5	(4, 10.7)	1683
2	10.7	(10.2, 15.8)	1685
3	15.6	(14.6, 26.6)	1690
4	26.2	(26.2, 34.9)	1701
5	35	(34.1, 38.4)	1709
6	38.5	(38, 41)	1713
7	40.5	(40, 44.9)	1715
8	44.8	(44.1, 47.8)	1719
9	48.2	(47.1, 58.9)	1723
10	59	(58.4, 64.7)	1733
11	64.7	(61.4, 73.8)	1739
12	73.7	(64.6, 79.6)	1748
13	79.5	(73.4, 93.2)	1754
14	92.5	(77.7, 99.7)	1767
15	99.6	(79.3, 106.8)	1774
16	106.6	(92.8, 109.9)	1781
17	109.8	(99.1, 113.8)	1784
18	113.6	(106.6, 120.7)	1788
19	120.1	(109.3, 124.5)	1795
20	124.2	(113.2, 126.9)	1799
21	126.7	(120.1, 130.8)	1801
22	130.6	(124.5, 134.9)	1805
23	134.5	(126.9, 141.5)	1809
24	141.4	(130.3, 145.4)	1816
25	145.9	(134.5, 150)	1820
26	149.6	(141, 157.8)	1824
27	156.5	(145.7, 159.8)	1831
28	159.5	(149.5, 161.7)	1834
29	162	(156.8, 170.9)	1836
30	170.4	(159.6, 178.8)	1845
31	172.7	(161.7, 179.4)	1847
32	178.9	(178.2, 192.8)	1853
33	192.8	(192.2, 199.4)	1867
34	199.5	(196.4, 210.6)	1874
35	219.5	(219.1, 220.9)	1894
36	226.6	(226, 226.8)	1901
37	231.3	(230.5, 231.8)	1906

Table 89: Posterior Estimates for s, conditioned on k = 37

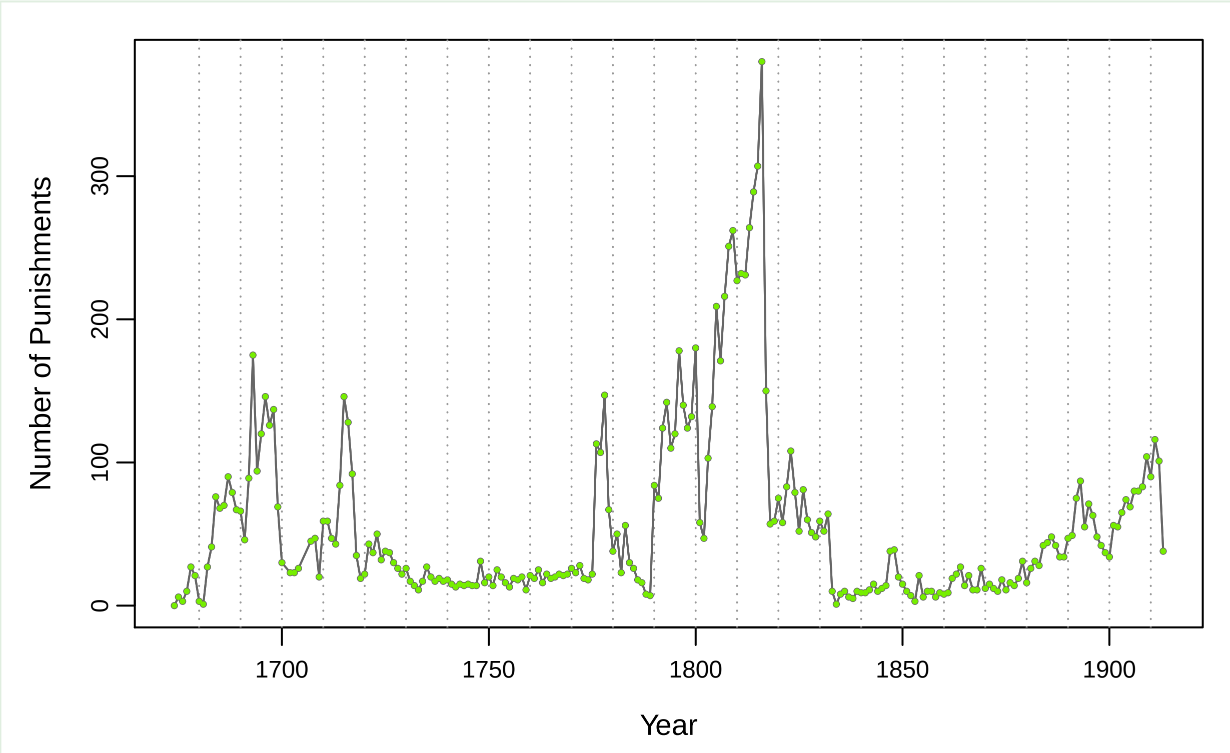


Figure 63: Number of Miscellaneous Punishments at the Old Bailey, counting by punishments per year (Old Bailey Online 2018b).

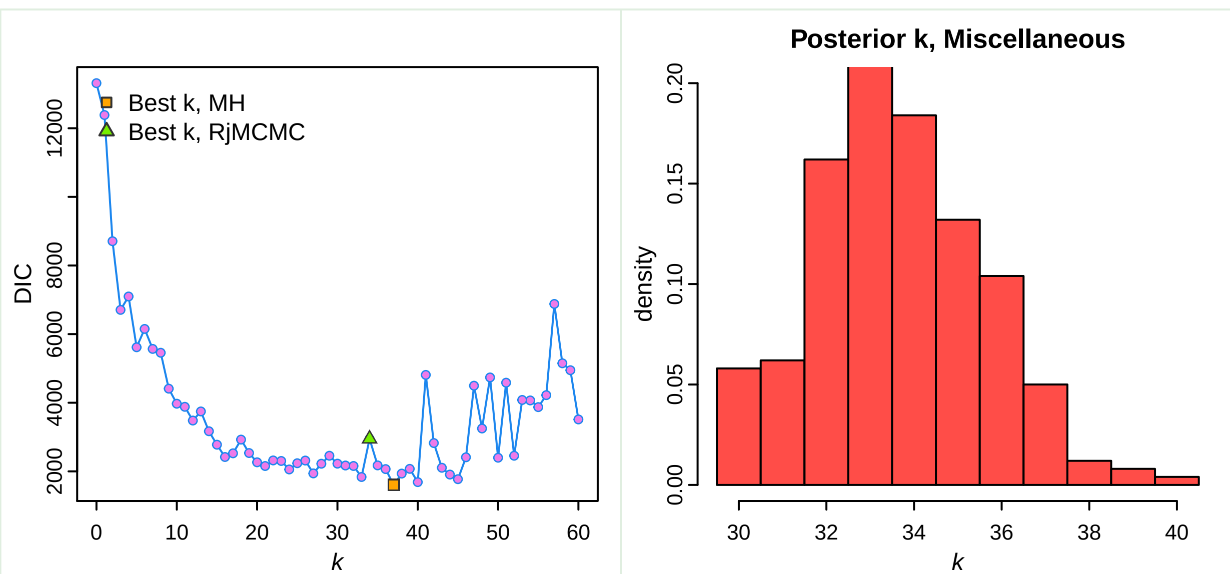


Figure 64: Model Estimates - DIC vs. Posterior k, Punishments by Offence, Miscellaneous (Old Bailey Online 2018b).

Figures and Tables - Punishments by Offence - Miscellaneous - MH - (Old Bailey Online 2018b)

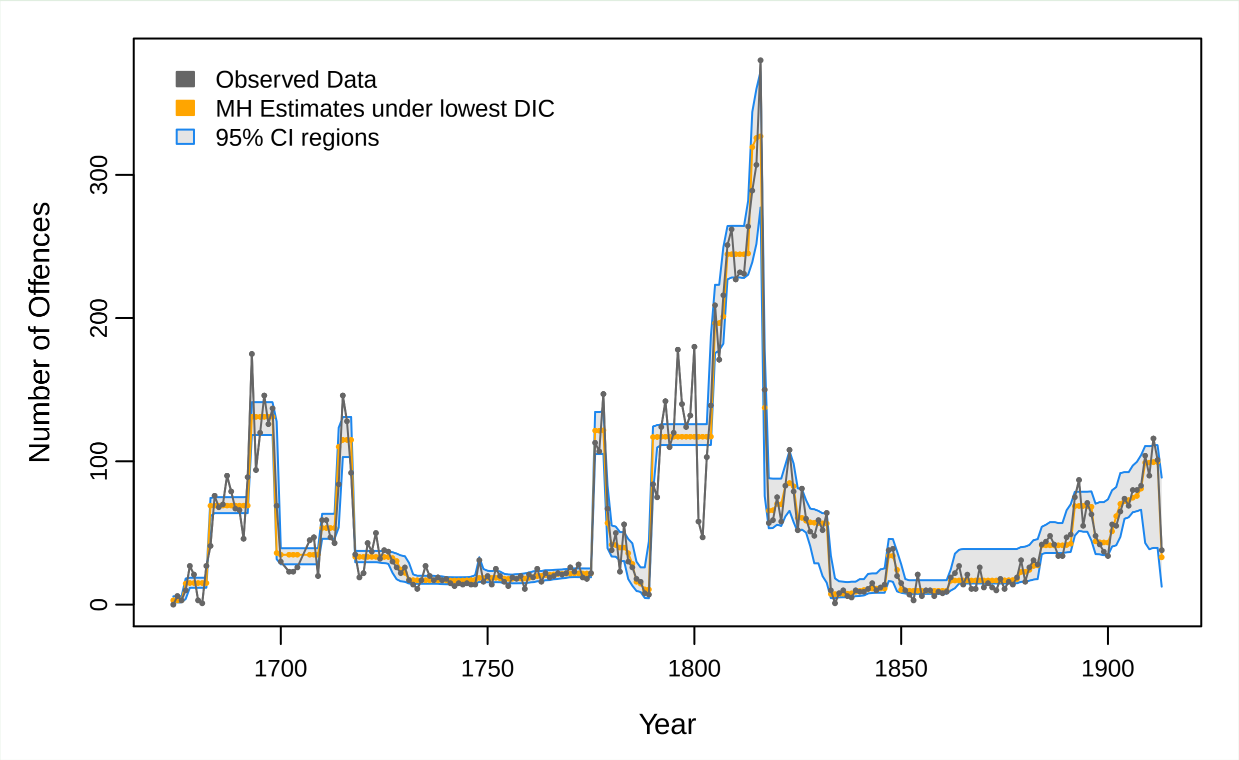


Figure 65: MH Estimates per Year, Punishments by Offence, Miscellaneous (Old Bailey Online 2018b).

k	DIC
0	13314.1
1	12385.3
2	8707.4
3	6706.5
4	7096.5
5	5616.6
6	6151.1
7	5568.5
8	5457.1
9	4409.9
10	3971.9
11	3881.4
12	3480.8
13	3746.3
14	3169.3
15	2777.2
16	2418.4
17	2525.5
18	2925.6
19	2533.8
20	2263.6
21	2156.0
22	2318.7
23	2302.5
24	2055.5
25	2236.7
26	2315.0
27	1940.5
28	2222.4
29	2454.9
30	2224.4
31	2170.4
32	2156.4
33	1836.1
34	2948.0
35	2174.0
36	2065.9
37	1604.2
38	1935.1
39	2073.0
40	1686.4
41	4811.2
42	2825.0
43	2104.6
44	1906.7
45	1772.3
46	2410.4
47	4495.9
48	3249.1
49	4737.3
50	2397.5
51	4584.1
52	2455.2
53	4081.1
54	4067.2
55	3872.0
56	4221.4
57	6879.2
58	5149.9
59	4947.7
60	3513.6

Table 90: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	3	(1.5, 5.8)
1	15.1	(11.8, 18.8)
2	69.1	(63.8, 74.9)
3	131.1	(118.6, 141.2)
4	34.8	(28.1, 39.3)
5	53.5	(46, 63.5)
6	115	(103, 131)
7	33.4	(29.6, 37.6)
8	18.7	(14.9, 32)
9	17.9	(14.4, 33)
10	18.1	(14.3, 23.6)
11	20.8	(16, 24.3)
12	115.7	(18.2, 132.7)
13	59.4	(20, 130.7)
14	41.2	(22, 129.6)
15	24.5	(10.8, 46.3)
16	10.6	(4.4, 78.2)
17	117.2	(111.4, 125.9)
18	196.5	(177.6, 224.1)
19	244.9	(230.2, 277.3)
20	326.9	(83, 372.3)
21	135.6	(74.1, 166.7)
22	62.4	(50.4, 104.6)
23	78.1	(34.2, 99.6)
24	55.4	(5, 63.7)
25	8	(4.7, 15)
26	12.3	(9, 42.9)
27	30.8	(10.1, 43.8)
28	9.7	(7.6, 17)
29	16.9	(14.7, 38.9)
30	27.9	(19.1, 54.4)
31	43.8	(37.7, 75)
32	66	(36.2, 79.4)
33	47.3	(35.3, 89.6)
34	75	(44.7, 109)
35	76.1	(13.8, 109.4)
36	32.8	(1.7, 107.7)
37	13.7	(1.4, 60.9)

Table 91: MH Posterior Estimates for h, conditioned on k = 37

s	Posterior Estimate	95% CI	Year
1	3.7	(3, 4.6)	1678
2	9.5	(9, 10)	1684
3	19.5	(19, 20)	1694
4	25.6	(25, 26.8)	1700
5	36.4	(36, 37)	1711
6	40.7	(40, 41.9)	1715
7	44.5	(44, 45)	1719
8	55.2	(51.7, 58.5)	1730
9	66.4	(56.1, 77.8)	1741
10	75.7	(66.4, 82)	1750
11	87.1	(74.6, 94.2)	1762
12	102.1	(86.2, 102.9)	1777
13	105.2	(90.8, 106)	1780
14	106.4	(102.3, 110.7)	1781
15	110.7	(105.2, 113.3)	1785
16	113.3	(111, 115.6)	1788
17	116.6	(116, 117.8)	1791
18	131.5	(131, 131.9)	1806
19	134.3	(133.3, 135)	1809
20	140.6	(140, 142.7)	1815
21	143.5	(143, 144)	1818
22	144.9	(144.1, 152)	1819
23	149	(146, 156.5)	1823
24	153.2	(150.4, 159.9)	1828
25	159.7	(159, 168.5)	1834
26	168	(162.3, 173.9)	1842
27	173.7	(172.3, 176)	1848
28	176.6	(175.1, 179.3)	1851
29	188.5	(187.5, 190.6)	1863
30	205.7	(201.2, 210.7)	1880
31	210.8	(208.5, 219.5)	1885
32	218.7	(216.8, 224.9)	1893
33	223.9	(222.4, 230.3)	1898
34	229.8	(227.1, 236.4)	1904
35	235.6	(229.2, 239.4)	1910
36	239.3	(234.5, 240.1)	1913
37	240	(239.1, 240.8)	1913

Table 92: MH Posterior Estimates for s, conditioned on k = 37

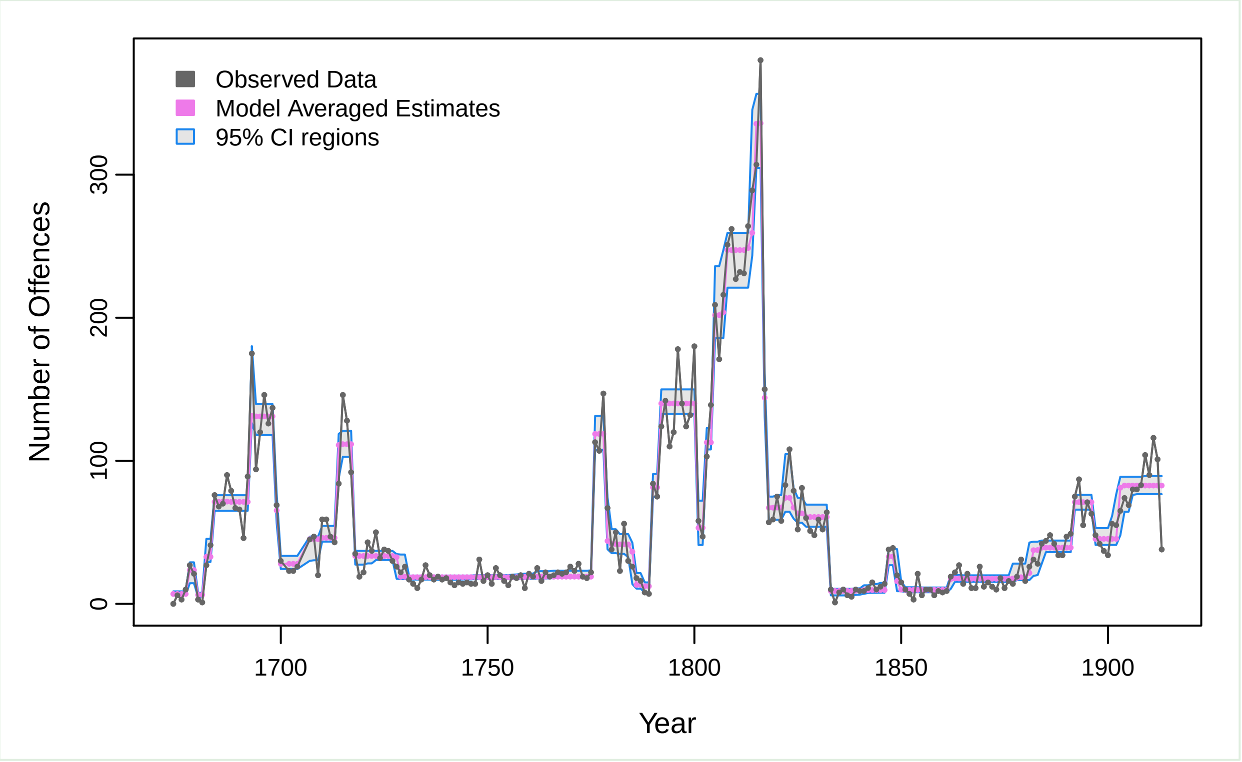


Figure 66: Model Averaged Estimates per Year, Punishments by Offence, Miscellaneous (Old Bailey Online 2018b).

k	Proportion
30	0.058
31	0.062
32	0.162
33	0.224
34	0.184
35	0.132
36	0.104
37	0.050
38	0.012
39	0.008
40	0.004

Table 93: Posterior estimate for k

h	Posterior Estimate	95% CI
0	6.8	(0, 8.5)
1	22.9	(0, 27.4)
2	6	(0, 7.7)
3	32.4	(0, 37.2)
4	70.7	(0, 74.1)
5	130.8	(0, 139.7)
6	65.3	(0, 86.4)
7	30.1	(0, 79.8)
8	45.2	(0, 56.8)
9	110.1	(0, 115.6)
10	33.4	(0, 109.9)
11	18.7	(0, 53.1)
12	107.8	(0, 119.8)
13	48.2	(0, 121.5)
14	35.4	(0, 119.8)
15	14.5	(0, 82.2)
16	81.9	(0, 148.8)
17	79.3	(0, 146.5)
18	53.9	(0, 141.6)
19	114.2	(0, 190.4)
20	190.4	(0, 259.3)
21	250.1	(0, 347.3)
22	154.1	(0, 352.6)
23	75	(0, 347.2)
24	71.5	(0, 154.1)
25	59.5	(0, 100.7)
26	9.6	(0, 59.7)
27	10.5	(0, 37.6)
28	9.9	(0, 36.5)
29	9.9	(0, 18.3)
30	19.9	(0, 39.3)
31	38.2	(0, 71.6)
32	70	(0, 76.2)
33	44.4	(0, 58.3)
34	82.1	(0, 88.9)

Table 94: Posterior Estimates for h, conditioned on k = 34

s	Posterior Estimate	95% CI	Year
1	4.4	(0, 5)	1679
2	6.5	(0, 6.7)	1681
3	8.1	(0, 9)	1683
4	10.3	(0, 10.9)	1685
5	19.5	(0, 19.9)	1694
6	25.3	(0, 25.9)	1700
7	26.3	(0, 26.9)	1701
8	33	(0, 36.6)	1707
9	40.4	(0, 41)	1715
10	44.2	(0, 44.8)	1719
11	55.2	(0, 58)	1730
12	102.3	(0, 102.9)	1777
13	105.2	(0, 105.9)	1780
14	106.5	(0, 112.9)	1781
15	112.4	(0, 116.9)	1787
16	116.5	(0, 118.9)	1791
17	118.7	(0, 127.5)	1793
18	127.1	(0, 130)	1802
19	129.3	(0, 131.5)	1804
20	131	(0, 134.1)	1805
21	134.1	(0, 141.5)	1809
22	141.6	(0, 143.6)	1816
23	143.7	(0, 144.6)	1818
24	144.6	(0, 151.2)	1819
25	148.7	(0, 159.5)	1823
26	159	(0, 173)	1833
27	167	(0, 175)	1841
28	173.7	(0, 179)	1848
29	176.9	(0, 188.8)	1851
30	188.8	(0, 208.4)	1863
31	209	(0, 218.6)	1883
32	218.6	(0, 223.8)	1893
33	223.7	(0, 227.7)	1898
34	229.2	(0, 230.2)	1904

Table 95: Posterior Estimates for s, conditioned on k = 34

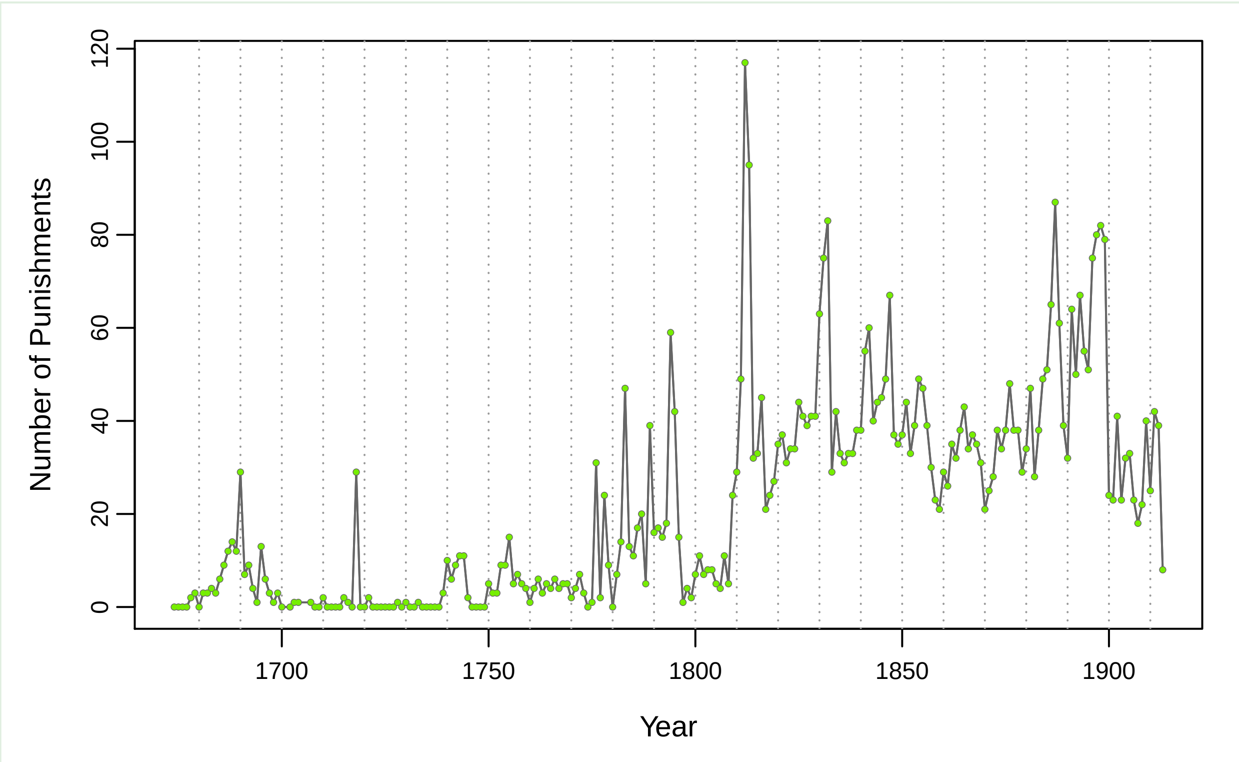
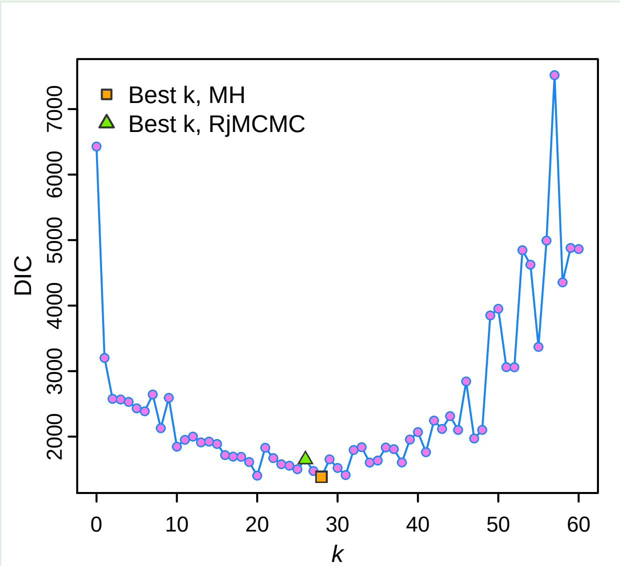


Figure 67: Number of Punishments at the Old Bailey, No Punishment, counting by punishments per year (Old Bailey Online 2018b).



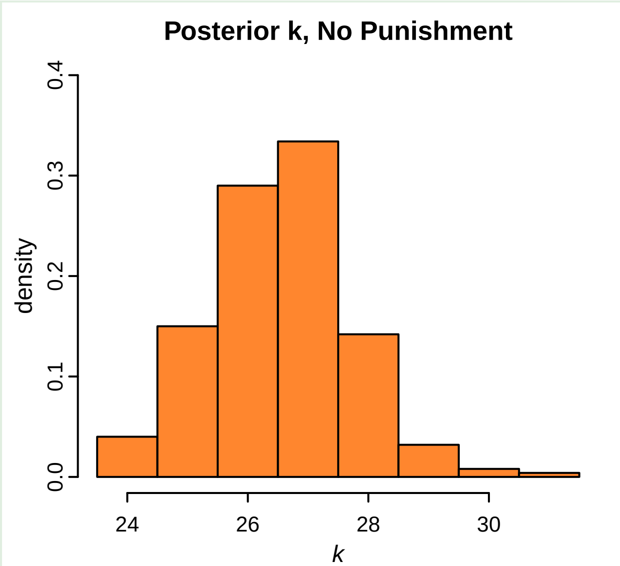


Figure 68: Model Estimates - DIC vs. Posterior k, Punishments by Offence, No Punishment (Old Bailey Online 2018b).

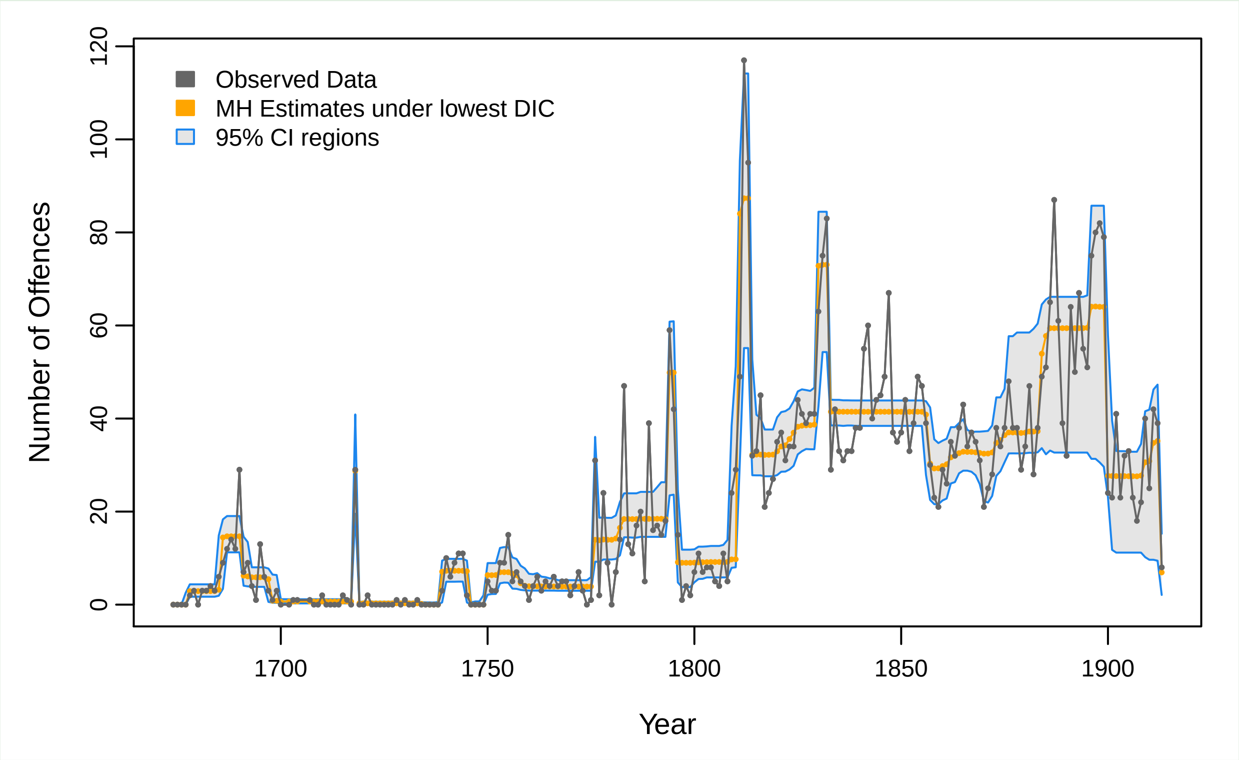


Figure 69: MH Estimates per Year, Punishments by Offence, No Punishment (Old Bailey Online 2018b).

k	DIC
0	6428.9
1	3201.4
2	2577.3
3	2567.3
4	2530.2
5	2432.2
6	2387.1
7	2643.0
8	2129.0
9	2592.6
10	1846.8
11	1951.1
12	2000.9
13	1912.3
14	1924.6
15	1888.3
16	1718.3
17	1695.3
18	1692.0
19	1613.1
20	1405.0
21	1831.0
22	1672.6
23	1579.5
24	1556.3
25	1502.7
26	1650.7
27	1475.3
28	1384.9
29	1653.7
30	1522.1
31	1413.8
32	1795.9
33	1839.1
34	1604.4
35	1635.0
36	1834.7
37	1809.5
38	1606.3
39	1956.1
40	2069.5
41	1762.3
42	2245.3
43	2117.7
44	2313.4
45	2102.2
46	2843.1
47	1971.9
48	2103.3
49	3850.6
50	3952.0
51	3060.6
52	3059.1
53	4846.0
54	4625.2
55	3369.9
56	4992.7
57	7517.9
58	4355.5
59	4880.4
60	4863.4

Table 96: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	0	(0, 0)
1	2.9	(1.7, 4.4)
2	14.7	(11.2, 19)
3	5.9	(3.8, 8)
4	0.7	(0.3, 1.2)
5	28.7	(19.5, 40.9)
6	0.3	(0.1, 0.6)
7	0	(0, 9)
8	6.8	(0, 9.7)
9	0	(0, 5.8)
10	7	(4.7, 12.4)
11	3.9	(3, 5.3)
12	14	(9.6, 36.1)
13	19.7	(14.9, 56.4)
14	48	(3.8, 60.9)
15	9.3	(6.2, 34.4)
16	85	(30, 103.4)
17	32.2	(27.6, 113.8)
18	38.7	(33.4, 46.6)
19	73.1	(54.3, 84.4)
20	41.5	(38.4, 43.9)
21	29.3	(21.6, 34.6)
22	32.8	(21.9, 44.9)
23	37.2	(32.7, 57.7)
24	59.4	(32.7, 66.2)
25	29.9	(22.6, 85.7)
26	28.3	(3.2, 42.9)
27	9.5	(0, 47.3)
28	2.9	(0, 13.9)

Table 97: MH Posterior Estimates for h, conditioned on k = 28

s	Posterior Estimate	95% CI	Year
1	4.4	(3.6, 5)	1679
2	12.4	(11.2, 13.6)	1687
3	17.6	(17, 19.6)	1692
4	24.6	(23.1, 27)	1699
5	44.4	(44, 44.9)	1719
6	45.7	(45, 46)	1720
7	61.7	(59.3, 66)	1736
8	65.7	(65, 72.9)	1740
9	72.6	(72, 76.9)	1747
10	76.7	(76, 79.8)	1751
11	85	(82.4, 90.5)	1759
12	102.4	(102, 103)	1777
13	109.1	(103.4, 120.8)	1784
14	120.6	(120, 123.5)	1795
15	122.8	(122, 135.5)	1797
16	137.4	(135.1, 138.6)	1812
17	140.5	(138.2, 141.2)	1815
18	148.8	(140.3, 153.5)	1823
19	156.5	(156, 157)	1831
20	159.6	(159, 160.1)	1834
21	183.5	(182.2, 184.9)	1858
22	190.9	(186.3, 202.3)	1865
23	200.1	(190.5, 207.2)	1875
24	210.8	(209.9, 213)	1885
25	226.2	(222, 227.8)	1901
26	235.1	(226, 239.4)	1910
27	239.2	(234.9, 239.9)	1913
28	239.9	(239.1, 240.8)	1913

Table 98: MH Posterior Estimates for s, conditioned on k = 28

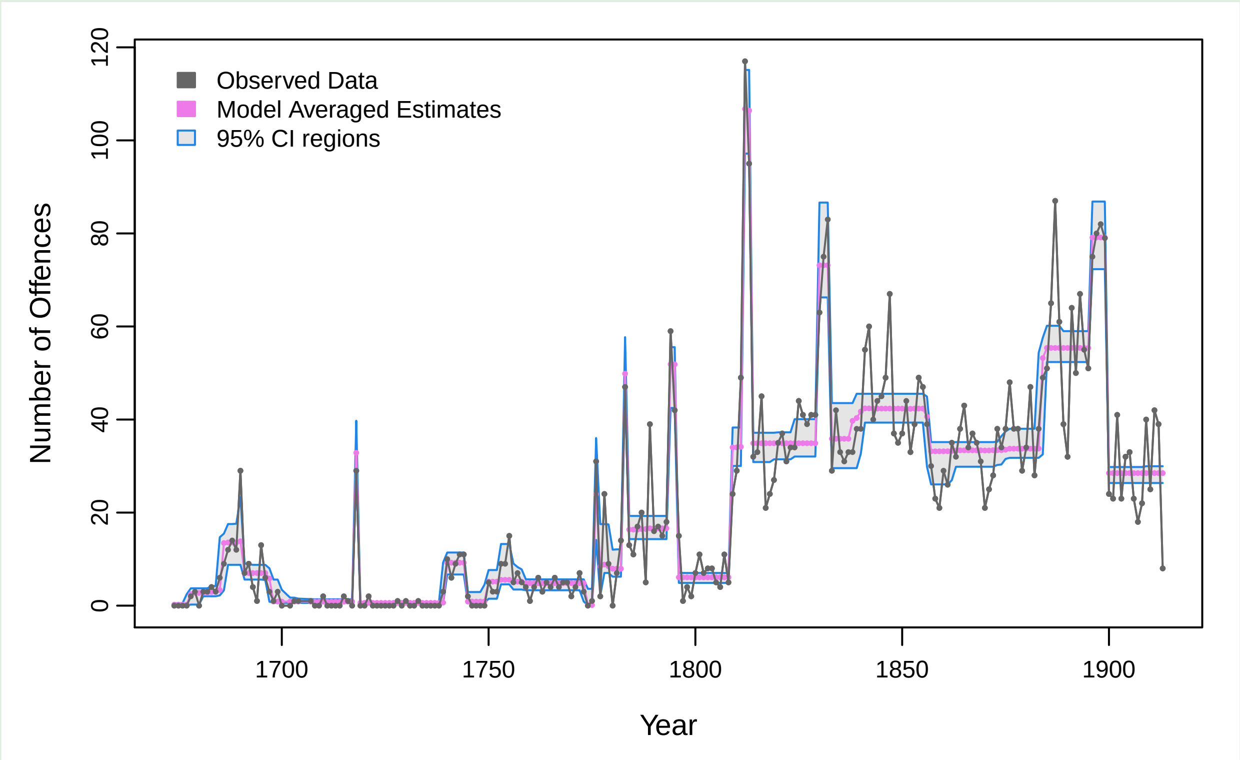


Figure 70: Model Averaged Estimates per Year, Punishments by Offence, No Punishment (Old Bailey Online 2018b).

k	Proportion
24	0.040
25	0.150
26	0.290
27	0.334
28	0.142
29	0.032
30	0.008
31	0.004

Table 99: Posterior estimate for k

h	Posterior Estimate	95% CI
0	0.2	(0, 0.5)
1	2.8	(0, 3.7)
2	13.6	(0, 16.6)
3	6.7	(0, 8.2)
4	0.9	(0, 6.1)
5	31.5	(0, 39.6)
6	0.6	(0, 34.2)
7	8.4	(0, 11.4)
8	0.8	(0, 11.4)
9	5.2	(0, 12.3)
10	0.1	(0, 9.1)
11	20.2	(0, 29.6)
12	7.9	(0, 35.3)
13	47.4	(0, 53.8)
14	16.2	(0, 57.5)
15	43.4	(0, 53.4)
16	6.1	(0, 54.4)
17	30.6	(0, 39.2)
18	101.5	(0, 115.1)
19	35.5	(0, 111.7)
20	67.4	(0, 79.1)
21	36.2	(0, 86.3)
22	41.8	(0, 45.5)
23	33.2	(0, 36.3)
24	55.4	(0, 59)
25	79.1	(0, 86.9)
26	28.1	(0, 30.3)

Table 100: Posterior Estimates for h, conditioned on k = 26

s	Posterior Estimate	95% CI	Year
1	4.4	(0, 7.4)	1679
2	12.1	(0, 13.9)	1687
3	17.4	(0, 22.6)	1692
4	24	(0, 38.2)	1698
5	44.3	(0, 45)	1719
6	45.4	(0, 45.9)	1720
7	65.8	(0, 67)	1740
8	71.3	(0, 71.9)	1746
9	76.4	(0, 79.7)	1751
10	100	(0, 100.9)	1774
11	102.2	(0, 102.9)	1777
12	103.5	(0, 106.9)	1778
13	109.2	(0, 109.9)	1784
14	110.2	(0, 111)	1785
15	120.1	(0, 121)	1795
16	122.1	(0, 123)	1797
17	135.2	(0, 135.9)	1810
18	138.2	(0, 138.9)	1813
19	140.2	(0, 140.8)	1815
20	156.2	(0, 156.9)	1831
21	159.1	(0, 159.8)	1834
22	164.5	(0, 183.5)	1839
23	183.2	(0, 190.5)	1858
24	210.9	(0, 211.9)	1885
25	222.5	(0, 223)	1897
26	226.7	(0, 226.9)	1901

Table 101: Posterior Estimates for s, conditioned on k = 26

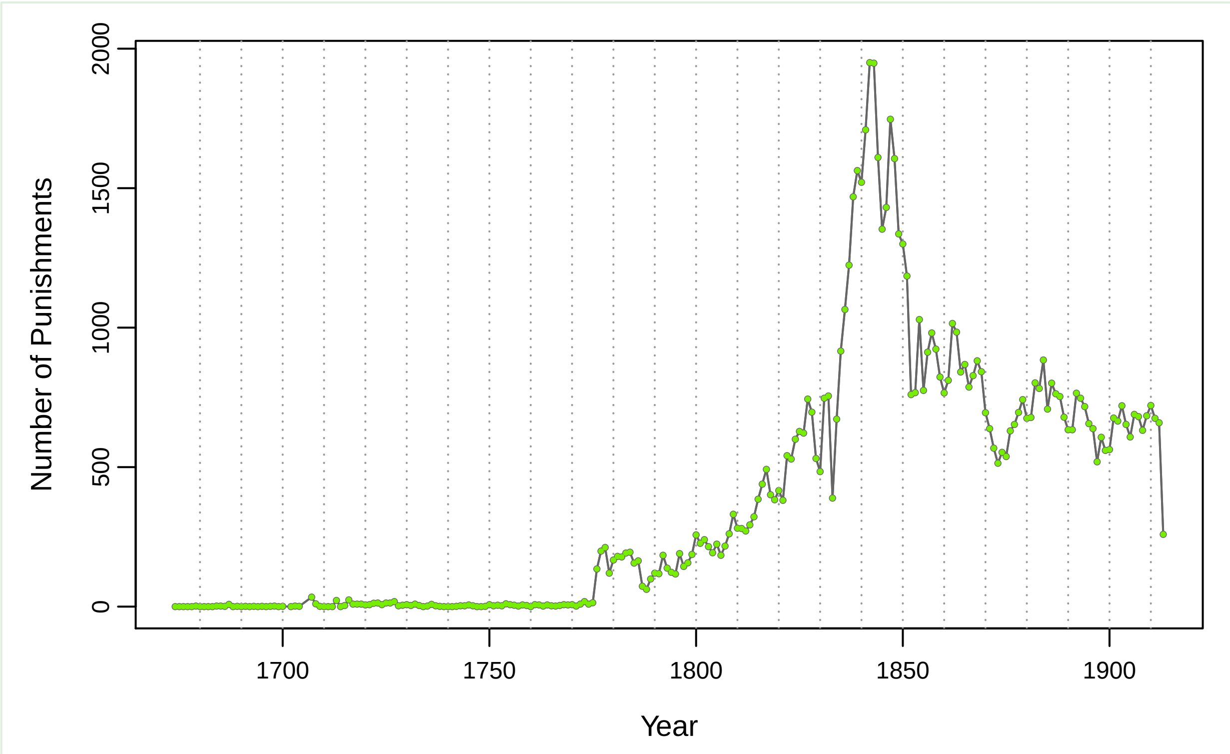


Figure 71: Number of Punishments at the Old Bailey, Imprisonment, counting by punishments per year (Old Bailey Online 2018b).

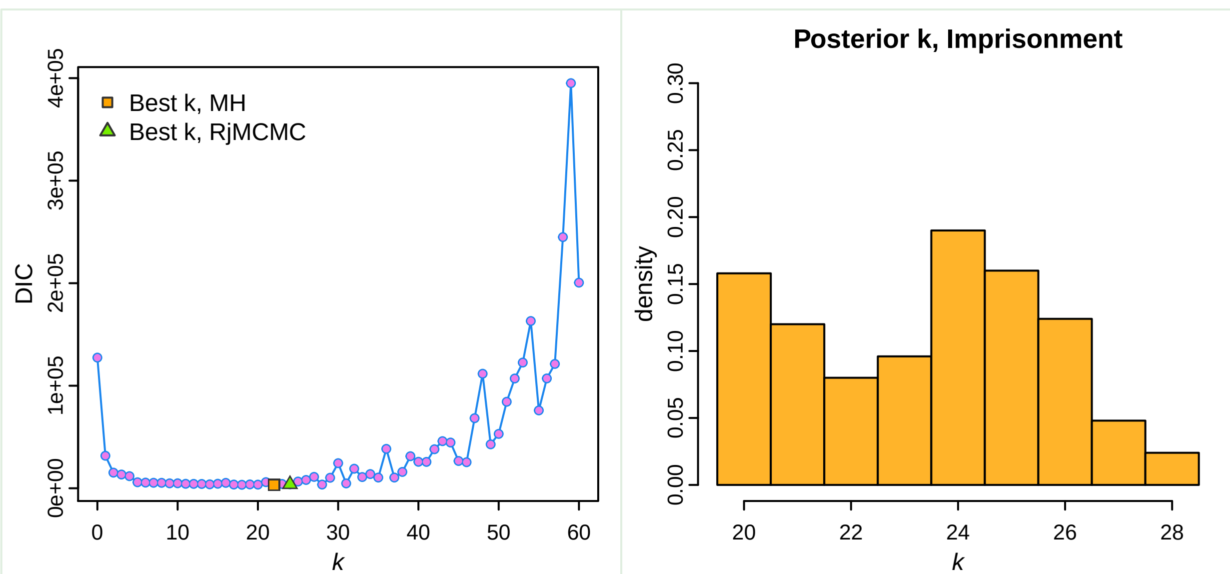


Figure 72: Model Estimates - DIC vs. Posterior k, Punishments by Offence, Imprisonment (Old Bailey Online 2018b).

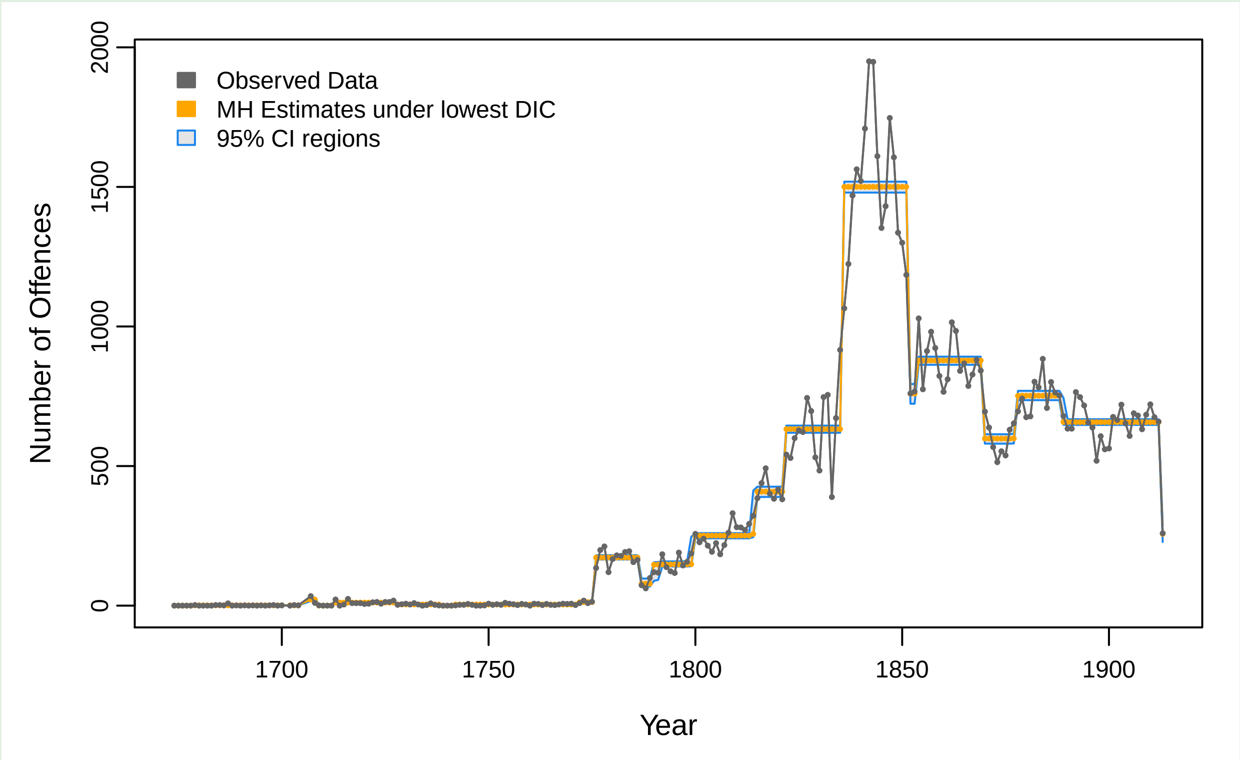


Figure 73: MH Estimates per Year, Punishments by Offence, Imprisonment (Old Bailey Online 2018b).

k	DIC
0	127411.8
1	31696.0
2	15229.3
3	13400.4
4	11796.3
5	5834.9
6	5507.4
7	5327.1
8	5286.2
9	4786.0
10	4834.7
11	4346.8
12	4257.6
13	4177.2
14	3817.0
15	4386.2
16	5262.0
17	3609.3
18	3387.6
19	3746.7
20	3541.1
21	5971.0
22	3231.5
23	4205.1
24	3836.8
25	6554.9
26	8098.4
27	11084.6
28	3592.6
29	10224.8
30	24447.9
31	4717.2
32	19044.1
33	10927.0
34	13830.6
35	10438.5
36	38408.0
37	10435.8
38	15916.6
39	31247.4
40	25812.1
41	25703.0
42	38062.3
43	45941.2
44	44585.3
45	26612.5
46	25397.8
47	68338.3
48	111683.1
49	42827.0
50	53014.1
51	84343.0
52	107048.9
53	122603.5
54	163109.5
55	75919.3
56	107114.7
57	121258.4
58	244966.7
59	395100.1
60	200528.1

Table 102: DIC for k, MH algorithm

h	Posterior Estimate	95% CI
0	0	(0, 0)
1	1	(0.7, 1.4)
2	21.7	(16.2, 28.6)
3	0.2	(0, 0.9)
4	11	(9.4, 12.8)
5	4.2	(3, 5.6)
6	0	(0, 0.6)
7	4	(3.4, 4.8)
8	12.5	(9.3, 16)
9	172.3	(165.2, 180.5)
10	78.4	(68.3, 97.1)
11	147.8	(140.1, 158.5)
12	250.8	(241.1, 259.7)
13	408.6	(389.3, 426.2)
14	632.5	(619.3, 645)
15	1500.3	(1479.7, 1518.8)
16	759.3	(723.3, 793.8)
17	878.2	(862.7, 892)
18	598.3	(580.3, 614)
19	751.9	(736.1, 769.5)
20	657.3	(647.3, 667.8)
21	256.9	(227.6, 286.8)
22	0	(0, 0)

Table 103: MH Posterior Estimates for h, conditioned on k = 22

s	Posterior Estimate	95% CI	Year
1	5.2	(3.2, 5.9)	1680
2	32.2	(31.1, 33.9)	1707
3	35.5	(35.1, 36)	1710
4	39.5	(39, 40)	1714
5	54.5	(54, 56)	1729
6	65.5	(64.2, 66)	1740
7	68.6	(68, 69.9)	1743
8	98.6	(98, 99.7)	1773
9	102.4	(102.1, 103)	1777
10	113.4	(113, 114)	1788
11	116.5	(116, 118.7)	1791
12	126.4	(125.5, 126.9)	1801
13	141.2	(140, 141.9)	1816
14	148.5	(148, 149)	1823
15	162.5	(162, 163)	1837
16	178.5	(178, 179)	1853
17	180.6	(180, 181)	1855
18	196.6	(196, 197)	1871
19	204.5	(204, 205)	1879
20	215.5	(215, 216.4)	1890
21	239.4	(239, 240)	1913
22	240.4	(240, 240.9)	1913

Table 104: MH Posterior Estimates for s, conditioned on k = 22

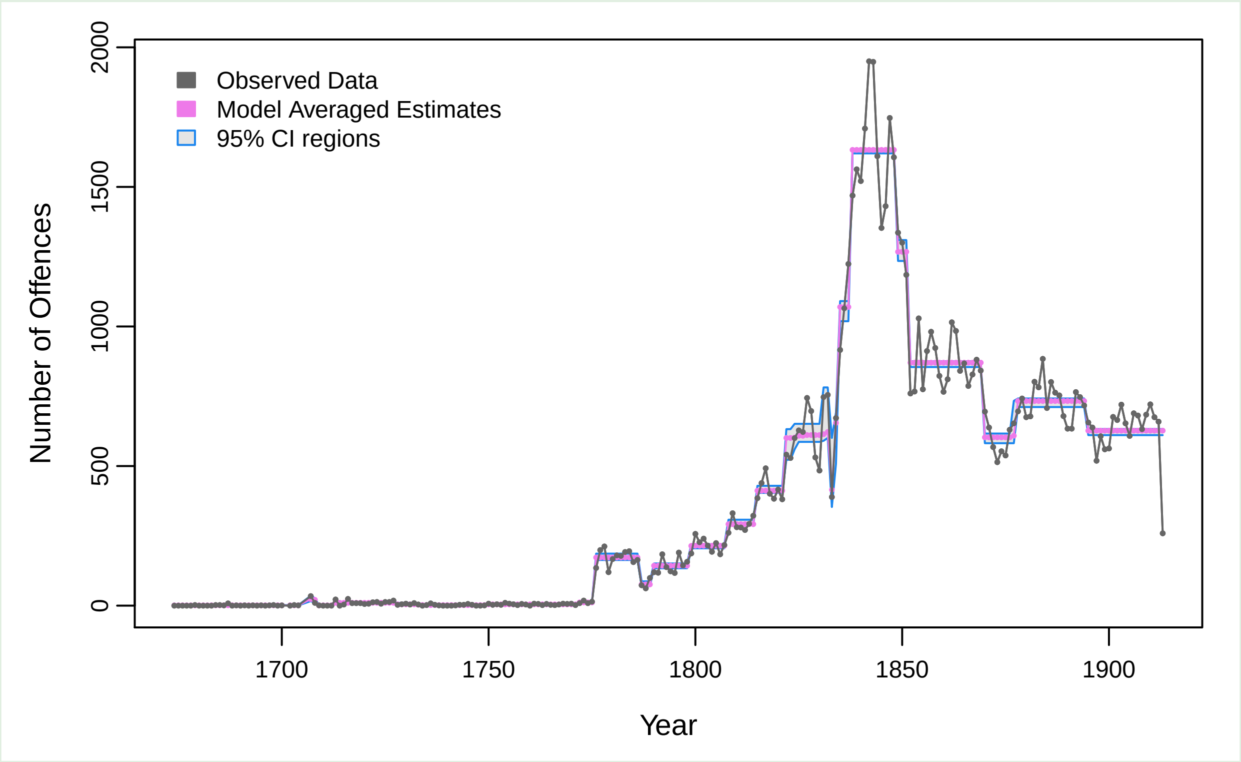


Figure 74: Model Averaged Estimates per Year, Punishments by Offence, Imprisonment (Old Bailey Online 2018b).

k	Proportion
20	0.158
21	0.120
22	0.080
23	0.096
24	0.190
25	0.160
26	0.124
27	0.048
28	0.024

Table 105: Posterior estimate for k

h	Posterior Estimate	95% CI
0	0.9	(0.2, 1.1)
1	16.5	(1.1, 32)
2	1.1	(0.6, 34.4)
3	10.9	(0.6, 19.2)
4	4.7	(0.8, 11.9)
5	2.3	(1.1, 11.4)
6	3.5	(1.2, 5.2)
7	4.9	(2.5, 11.7)
8	12.8	(4.9, 174.1)
9	82.8	(9.8, 174.1)
10	138.5	(69, 174.1)
11	151	(79.1, 219.1)
12	218.3	(143.1, 304.2)
13	292.1	(217.8, 429.3)
14	424.4	(301.3, 599.5)
15	624.2	(405.8, 778.4)
16	383.3	(354.6, 614.1)
17	655.8	(508.6, 692.4)
18	1069.8	(1018.8, 1090.9)
19	1632.6	(1620.3, 1632.6)
20	1278.9	(1243.5, 1286)
21	870.4	(862, 874.2)
22	606	(581.9, 608.7)
23	733.5	(720.3, 735.5)
24	626.9	(619.1, 632.5)

Table 106: Posterior Estimates for h, conditioned on k = 24

s	Posterior Estimate	95% CI	Year
1	32	(4.3, 33.9)	1706
2	35	(32, 35.8)	1709
3	39.1	(34.1, 39.9)	1714
4	54	(35.5, 54.9)	1728
5	59.1	(39.3, 64.4)	1734
6	76.1	(54.6, 76.9)	1751
7	83.8	(63.6, 99.3)	1758
8	98.6	(75.5, 102.9)	1773
9	102.8	(98.1, 113.7)	1777
10	113.7	(102.5, 116.7)	1788
11	116.7	(113.6, 125.7)	1791
12	125.7	(116.9, 135)	1800
13	134.8	(125, 142)	1809
14	141.6	(134.7, 148.7)	1816
15	148.7	(141.6, 158.6)	1823
16	159.5	(148.3, 159.8)	1834
17	160.6	(159.6, 160.9)	1835
18	161.5	(161, 161.9)	1836
19	164.6	(164, 164.9)	1839
20	175.4	(175.2, 175.9)	1850
21	178.6	(178.1, 178.9)	1853
22	196.7	(196.1, 197)	1871
23	204.5	(203.8, 204.9)	1879
24	221.7	(221, 222.5)	1896

Table 107: Posterior Estimates for s, conditioned on k = 24

7 Appendix: R Code

```r
# January 2020
# by Julianne Shields and Elena Moltchanova

### MCMC fixed k

# Crime / Punishment chosen before analysis, commenting out relevant line:
for(z in 0:9) { # Loop over z - Crime
for(z in 0:6) { # Loop over z - Punishment

rm(list=ls())
# library(truncnorm)

subset_num <- z
# This is the number for folders

subset_index <- subset_num + 2
# This is the index for the large dataframe

data_prefix <- as.character(label_key[1,(subset_num + 1)])
# This is the label for the filenames / object names

full_data_name <- as.character(label_key[2,(subset_num + 1)])
# Name to be appended to plots

label_path <- paste(subset_num, "_", data_prefix, "/", sep = "")
type_prefix <- "fixed/"
directory_prefix <- paste(directory_prefix0, label_path, type_prefix, sep = "")
# This is the full prefix for the folder where things are saved

count_total <- as.numeric(data_in[1,subset_index])
y_year <- as.numeric(as.character(data_in_cut[,1]))
y_dat <- data_in_cut[,subset_index]
dat <- cbind(y_year, y_dat)

data_label_prefix <- paste(directory_prefix0,
                           label_path, "data_", data_prefix, ".csv", sep = "")
write.csv(dat, file = data_label_prefix)

#############################
#
#     Setting up dataspace
#
#############################

k_max <- 60

ITER <- 50000
burn <- 20000
thinned.size <- 500

site.sd <- 1
```

```r
a.h <- 0.01
b.h <- 0.01

reference_file <- array(0, dim = c(2, 9))
reference_file[1,] <- (c("data_prefix", "full_data_name",
                         "ITER", "burn", "thinned.size", "k_max",
                         "site.sd", "a.h", "b.h"))
reference_file[2,] <- (c(data_prefix, full_data_name,
                         ITER, burn, thinned.size, k_max,
                         site.sd, a.h, b.h))

ref_file_prefix <- paste(directory_prefix, "reference_file.csv", sep = "")
write.csv(reference_file, file = ref_file_prefix)

rm(list = c("data_label_prefix", "subset_num", "subset_index", "label_key",
            "data_in", "data_in_cut", "directory_prefix0", "type_prefix",
            "label_path", "ref_file_prefix"))

#############################
#
#     The Sampler
#
#############################

Time <- dat[,1]

DIC <- array(dim=c((k_max + 1),2))

L <- length(y_dat)

start_time <- Sys.time()

# set number of breaks
for(k in 0:k_max){

  k_iter <- k + 1

seq_iter <- seq(from = (burn + 1), to = ITER)
thinned_iter <- sort(sample(seq_iter, size = thinned.size))

# prepare monitors (heights and sites)
MON.h <- array(dim = c(thinned.size, (k_iter)))
MON.s <- array(dim =c(thinned.size, (k_iter + 1)))
MON.z <- array(dim = c(thinned.size, (length(y_dat))))

# and for the dic
mon.lik <- numeric(ITER)

x <- Time - min(Time)+1

# initialize
sites <- seq(0,max(x)+1,length.out=(k+2))
z.alloc <- as.numeric(cut(x,sites))
```

```r
heights <- tapply(y_dat,z.alloc,mean)

###

for(iter in 1:ITER){

# heights (all together)
heights <- rgamma(k+1,a.h+tapply(y_dat,z.alloc,sum),b.h+c(table(z.alloc)))+(a.h*b.h)

if (k>0){
# sites (choose one)
which.site <- sample(1:k,size=1)

# sample a uniform value
sites.new <- sites
sites.new[which.site+1] <- rtruncnorm(1,a=sites.new[which.site],
            b=sites.new[which.site+2],mean=sites.new[which.site+1],
            sd=site.sd)
z.alloc.new <-  as.numeric(cut(x,sites.new))

# rejection ratio
logR <- sum(dpois(y_dat,heights[z.alloc.new],log=T))- # likelihood
        sum(dpois(y_dat,heights[z.alloc],log=T))+
    sum(log(sites.new[-1]-sites.new[-(k+2)])) - #prior
    sum(log(sites[-1]-sites[-(k+2)]))+ # proposal
    log(dtruncnorm(sites[which.site+1],a=sites.new[which.site],
            b=sites.new[which.site+2],mean=sites.new[which.site+1],
            sd=site.sd))-
    log(dtruncnorm(sites.new[which.site+1],a=sites[which.site],
            b=sites[which.site+2],mean=sites[which.site+1],
            sd=site.sd))

logU <- log(runif(1,0,1))

if(logU < logR){
    sites <- sites.new
    z.alloc <- z.alloc.new}
}

# monitor update

# MON.h[iter,] <- heights
# MON.s[iter,] <- sites
# MON.z[iter,] <- z.alloc

if (iter %in% thinned_iter) {
  save_index <- which(thinned_iter == iter)
  print(paste("k = ", k, " ", save_index))
  MON.h[save_index,] <- heights
  MON.s[save_index,] <- sites
  MON.z[save_index,] <- z.alloc}

# and we need DIC
```

```
mon.lik[iter] <- sum(dpois(y_dat,heights[z.alloc],log=T))}


if (k == 0){
  y.est <- array(dim=c(thinned.size,L))
  for(j in 1:thinned.size){
    y.est[j,] <- MON.h[j][MON.z[j,]]}
  y.mn <- apply(y.est,2,mean)} else {
    y.est <- array(dim=c(thinned.size,L))
    for(j in 1:thinned.size){
      y.est[j,] <- MON.h[j,][MON.z[j,]]}
    y.mn <- apply(y.est,2,mean)}

print(paste(k, " complete"))

s.mean <- apply(MON.s,2,mean)
if(k>0){h.mean <- apply(MON.h,2,mean)}else{
  h.mean <- mean(MON.h)}
z.alloc.mean <- as.numeric(cut(x,s.mean))
lik.mean <- sum(dpois(y_dat,h.mean[z.alloc.mean],log=T))

D.bar <- -2*mean(mon.lik[(burn+1):ITER])
bar.D <- -2*lik.mean
p.D <- D.bar-bar.D
DIC[k_iter,1] <- k
DIC[k_iter,2] <- D.bar+p.D

} #end of DIC

end_time <- Sys.time()

# } # end of crime

### RjMCMC Crime/Punishment

# Crime / Punishment chosen before analysis, commenting out relevant line:
for(z in 0:9) { # Loop over z - Crime
for(z in 0:6) { # Loop over z - Punishment

rm(list=ls())
library(truncnorm)

subset_num <- z

subset_index <- subset_num + 2
# This is the index for the large dataframe

data_prefix <- as.character(label_key[1,(subset_num + 1)])
# This is the label for the filenames / object names

full_data_name <- as.character(label_key[2,(subset_num + 1)])
# Name to be appended to plots
```

```r
label_path <- paste(subset_num, "_", data_prefix, "/", sep = "")
type_prefix <- "BD/"
directory_prefix <- paste(directory_prefix0, label_path, type_prefix, sep = "")
# This is the full prefix for the folder where things are saved

count_total <- as.numeric(data_in[1,subset_index])
y_year <- as.numeric(as.character(data_in_cut[,1]))
y_dat <- data_in_cut[,subset_index]
dat <- cbind(y_year, y_dat)

data_label_prefix <- paste(directory_prefix0,
                           label_path, "data_", data_prefix, ".csv", sep = "")

#############################
#
#     Setting up algorithm
#
#############################

k_max <- 100

ITER <- 50000
burn <- 20000
thinned.size <- 500

site.sd <- 1
a.h <- 0.01
b.h <- 0.01

k_init <- 3

reference_file <- array(0, dim = c(2, 10))
reference_file[1,] <- (c("data_prefix", "full_data_name",
                         "ITER", "burn", "thinned.size", "k_max",
                         "site.sd", "a.h", "b.h", "k_init"))
reference_file[2,] <- (c(data_prefix, full_data_name,
                         ITER, burn, thinned.size, k_max,
                         site.sd, a.h, b.h, k_init))

ref_file_prefix <- paste(directory_prefix, "reference_file.csv", sep = "")
write.csv(reference_file, file = ref_file_prefix)

rm(list = c("data_label_prefix", "subset_num", "subset_index", "label_key",
            "data_in", "data_in_cut", "directory_prefix0", "type_prefix",
            "label_path", "ref_file_prefix"))

#############################
#
#     The Sampler
#
#############################

Time <- dat[,1]
```

```
L <- length(y_dat)

# choosing prior distribution for k
prior.k <- function(k){1/(k_max+1)*(k<=k_max)*(k>=0)}
# prepare monitors (heights and sites)
MON.h <- array(0, dim=c(ITER, (k_max+1)))
MON.bp <- array(0, dim=c(ITER, (k_max + 2)))
MON.z <- array(dim = c(ITER, L))
MON.lik <- array(dim = c(ITER, 2))

# and monitoring move type
x <- Time - min(Time)+1

# initialize
bp_init <- seq(0,max(x)+1,length.out=(k_init+2))
z_init <- as.numeric(cut(x,bp_init))
h_init <- tapply(y_dat,z_init,mean)

bp_vec <- bp_init
z_alloc <- z_init
h_vec <- h_init
k <- k_init

# evaluating a vector of possible p.b and p.d beforehand
p.b <- function(k){
  if(k==k_max){p.b <- 0}else{
    if(k==0){p.b <- 0.9}else{p.b <- 0.45}}
  return(p.b)}

p.d <- function(k){
  if(k==k_max){p.d <- 0.9}else{
    if(k==0){p.d <- 0}else{p.d <- 0.45}}
  return(p.d)}

p.h <- function(k){
  if(k==0){p.h <- 0.1}else{
    p.h <- 0.05}
  return(p.h)}

p.s <- function(k){
  if(k==0){p.s <- 0}else{
    p.s <- 0.05}
  return(p.s)}

# start_time <- Sys.time()

for(iter in 1:ITER){
  probs <- c(p.b(k), p.d(k), p.h(k), p.s(k))
  step_choice <- sample(x=c("b", "d", "h", "s"), size = 1, prob = probs)

if(step_choice=='h'){
  ### SAMPLING A HEIGHT (H):
```

```
  h_vec_old <- h_vec
  int_sample <- sample.int((k+1),1)
  h_j <- h_vec[int_sample]
  temp <- runif(n=1, min=(-0.1), max=(0.1))
  h_j_prime <- (h_j * (exp(temp)))

  h_vec_new <- h_vec
  h_vec_new[int_sample] <- h_j_prime

  log_likelihood_old <- sum(dpois(y_dat,h_vec_old[z_alloc],log=T))
  log_likelihood_new <- sum(dpois(y_dat,h_vec_new[z_alloc],log=T))

  logR <- log_likelihood_new-log_likelihood_old+
    sum(dgamma(h_j_prime,a.h,b.h,log=T))-
    sum(dgamma(h_j,a.h,b.h,log=T))+
    # proposal ratio
    log(h_j_prime)-log(h_j)

  logU <- log(runif(1,0,1))

  if (logU < logR) {
    bp_vec <- bp_vec
    h_vec <- h_vec_new
    lik <- log_likelihood_new
    z_alloc <- z_alloc} else {
    bp_vec <- bp_vec
    h_vec <- h_vec_old
    lik <- (log_likelihood_old)
    z_alloc <- z_alloc}

  MON.bp[iter, 1:(k+2)] <- bp_vec
  MON.h[iter, 1:(k+1)] <- h_vec
  MON.lik[iter,1] <- k
  MON.lik[iter,2] <- lik
  MON.z[iter, 1:L] <- z_alloc


} # end of move.type H

if(step_choice=='s'){
  bp_vec_old <- bp_vec
  z_alloc_old <- z_alloc

  int_sample <- sample.int(k,1)

  bp_j_minus_1 <- bp_vec[int_sample]
  bp_j_plus_1 <- bp_vec[int_sample + 2]
  bp_j <- bp_vec[int_sample + 1]
  bp_j_star <- (rtruncnorm(1, a=bp_j_minus_1, b=bp_j_plus_1, mean = bp_j,
                          sd = site.sd))
  bp_vec_new <- bp_vec
  bp_vec_new[int_sample + 1] <- bp_j_star
```

```r
  z_alloc_new <- as.numeric(cut(x, bp_vec_new))

  log_likelihood_old <- sum(dpois(y_dat,h_vec[z_alloc_old],log=T))
  log_likelihood_new <- sum(dpois(y_dat,h_vec[z_alloc_new],log=T))

  # rejection ratio
  logR <- log_likelihood_new- # likelihood
    log_likelihood_old+
    sum(log(bp_vec_new[-1]-bp_vec_new[-(k+2)])) - #prior
    sum(log(bp_vec_old[-1]-bp_vec_old[-(k+2)]))+  # proposal
    log(dtruncnorm(bp_j, a = bp_j_minus_1, b = bp_j_plus_1,
                   mean = bp_j_star, sd = site.sd))-
    log(dtruncnorm(bp_j_star, a = bp_j_minus_1, b = bp_j_plus_1,
                   mean = bp_j, sd = site.sd))
  logU <- log(runif(1,0,1))

  if (logU < logR) {
    bp_vec <- bp_vec_new
    h_vec <- h_vec
    lik <- log_likelihood_new
    z_alloc <- z_alloc_new} else {
    bp_vec <- bp_vec_old
    h_vec <- h_vec
    lik <- log_likelihood_old
    z_alloc <- z_alloc_old}

  MON.bp[iter, 1:(k+2)] <- bp_vec
  MON.h[iter, 1:(k+1)] <- h_vec
  MON.lik[iter,1] <- k
  MON.lik[iter,2] <- lik
  MON.z[iter, 1:L] <- z_alloc

}

if(step_choice=='b'){
  ### BIRTH MOVE

  h_vec_old <- h_vec
  bp_vec_old <- bp_vec
  z_alloc_old <- z_alloc

  s_star <- runif(1,0,L)
  u <- runif(1,0,1)

  j <- which(bp_vec>s_star)[1]-1
  bp_vec_new <- c(bp_vec[1:j], s_star, bp_vec[(j+1):(k+2)])
  z_alloc_new <- as.numeric(cut(x,bp_vec_new))

  # height left
  h.L <- exp(log(h_vec[j]) - (bp_vec[j+1] - s_star)/
               (bp_vec[j+1] - bp_vec[j])*log((1-u)/u))
  # height to the right
  h.R <- exp(log(h_vec[j]) + (s_star-bp_vec[j])/
```

```
              (bp_vec[j+1] - bp_vec[j])*log((1-u)/u))

  if(j==1){h_vec_new <- c(h.L,h.R,h_vec[-1])}else{
    if(j==k+1){h_vec_new <- c(h_vec[1:k],h.L,h.R)}else{
      h_vec_new <- c(h_vec[1:(j-1)],h.L,h.R,h_vec[(j+1):(k+1)])
    }}

  log_likelihood_old <- sum(dpois(y_dat,h_vec_old[z_alloc_old],log=T))
  log_likelihood_new <- sum(dpois(y_dat,h_vec_new[z_alloc_new],log=T))

  logR <-
    # likelihood
    log_likelihood_new-log_likelihood_old+
    # priors
    log(prior.k(k+1))-log(prior.k(k))+
    log(2)+log(k+1)+log(2*k+3)-2*log(L)+
    log(s_star-bp_vec_old[j])+log(bp_vec_old[j+1]-s_star)-
    log(bp_vec_old[j+1]-bp_vec_old[j])+
    (dgamma(h.R,a.h,b.h,log=T))+(dgamma(h.L,a.h,b.h,log=T))-
    (dgamma(h_vec_old[j],a.h,b.h,log=T))+
    # proposal ratio
    log(p.d(k+1))+log(L)-log(p.b(k))-log(k+1)+
    # Jacobian
    2*log(h.L+h.R)-log(h_vec_old[j])

  logU <- log(runif(1,0,1))

  if (logU < logR) {
    bp_vec <- bp_vec_new
    h_vec <- h_vec_new
    lik <- log_likelihood_new
    z_alloc <- z_alloc_new
    k <- k+1} else {
    bp_vec <- bp_vec_old
    h_vec <- h_vec_old
    lik <- log_likelihood_old
    z_alloc <- z_alloc_old
    k <- k}

  MON.bp[iter, 1:(k+2)] <- bp_vec
  MON.h[iter, 1:(k+1)] <- h_vec
  MON.lik[iter,1] <- k
  MON.lik[iter,2] <- lik
  MON.z[iter, 1:L] <- z_alloc

} # end of move.type B

if(step_choice=='d'){
  ### DEATH MOVE

  h_vec_old <- h_vec
  bp_vec_old <- bp_vec
  z_alloc_old <- z_alloc
```

```r
j_range <- c(seq.int(from = 2, to = (k+1)))
if (k == 1){
  j <- 2}else{
    j <- sample(j_range, size = 1)}

bp_vec_new <- bp_vec_old[-j]

h_j_prime <- exp(((bp_vec_old[j]-bp_vec_old[j-1])*
                    log(h_vec_old[j-1])+
                    (bp_vec_old[j+1]-bp_vec_old[j])*
                    log(h_vec_old[j]))/(
                      bp_vec_old[j+1]-bp_vec_old[j-1]))

z_alloc_new <- as.numeric(cut(x, bp_vec_new))

if(k == 1){h_vec_new <- h_j_prime}else{
  if(j == 2){h_vec_new <- c(h_j_prime, h_vec_old[3:(k+1)])}else{
    if(j == (k+1)){h_vec_new <- c(h_vec_old[1:(k-1)],h_j_prime)}else{
      h_vec_new <- c(h_vec_old[1:(j-2)],h_j_prime,h_vec_old[(j+1):(k+1)])
    }}}

log_likelihood_old <- sum(dpois(y_dat,h_vec_old[z_alloc_old],log=T))
log_likelihood_new <- sum(dpois(y_dat,h_vec_new[z_alloc_new],log=T))

logU <- log(runif(1,0,1))

logR <- # likelihood
  log_likelihood_new-log_likelihood_old+
  # priors
  log(prior.k(k))-log(prior.k(k-1))+
  2*log(L)-log(2*k+1)-log(2*k)+
  log(bp_vec_old[j+1]-bp_vec_old[j-1])-
  log(bp_vec_old[j+1]-bp_vec_old[j])-
  log(bp_vec_old[j]-bp_vec_old[j-1])+
  sum(dgamma(h_vec_new,a.h,b.h,log=T))-
  sum(dgamma(h_vec_old,a.h,b.h,log=T))+
  # proposal ratio
  log(p.b(k-1))+log(k)-log(p.d(k))-log(L)+
  # Jacobian
  log(h_j_prime)-2*log(h_vec_old[j-1]+h_vec_old[j])

if (logU < logR) {
  bp_vec <- bp_vec_new
  h_vec <- h_vec_new
  lik <- log_likelihood_new
  z_alloc <- z_alloc_new
  k <- k-1} else {
  bp_vec <- bp_vec_old
  h_vec <- h_vec_old
  lik <- log_likelihood_old
  z_alloc <- z_alloc_old
  k <- k}
```

```
  MON.bp[iter, 1:(k+2)] <- bp_vec
  MON.h[iter, 1:(k+1)] <- h_vec
  MON.lik[iter,1] <- k
  MON.lik[iter,2] <- lik
  MON.z[iter, 1:L] <- z_alloc

} # end of move.type D

} #end of iterations

print(paste(full_data_name, "Simulation finished"))
} # End of loop over z
```

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